A TREATISE OF THE Natural Grounds, AND PRINCIPLES OF HARMONY.

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ERRA-

ERRATA, &c.

PAg. 1. lin. 13. read *Medium*. *P.* 20. *l.* 19. *r.* and Contrary. Quadruple. *P.* 23. *l.* 15. *for* do, *r.* no. *P.* 28. *l.* 20. *r.* Shaflaunces. *P.* 42. *l.* 19. *r.* Ridged. *P.* 64. *l.* 14. *for* recourſe, *r.* courſe. *P.* 99. *l.* 24. *for* 28, *r.* 18. *P.* 108. *l.* 13. *for* Rations, or *r.* Rations. Or if. *P.* 111. *l.* 21. *after* Progreſſion, *add* (Underſtanding, together with the Ratio's, the Intervals themſelves, as is before premiſed) *P.* 127. *l.* 10. *for* on, *r.* or. *P.* 132. *l.* 3. *for* it, *r.* it. *P.* 139. *l.* 9. *for* that was to expreſs it. *r.* that was ſuppoſed to be of the deepeſt ſetled Pitch in Nature, and adapted ſteely to expreſs it. *P.* 148. *l.* 2. *r.* Degrees. *P.* 152. *l.* 3. *r.* gives. *P.* 156. *l.* 8. *for* Proper, *r.* with a Flat 6th. *Ibid. l.* 20. *for* the Fourth, *r.* the Second, Fourth. P. 157. l. 3. for *Minor* r. *Major*. *Ibid. l.* 6. *Dele.* Third. *P.* 159. *l.* 19. *for* no one, *r.* but one. P. 160. Under *Scale* IV. Degr. 4th, for *Minor*, r. *Major*. Ibid. Under *Scale* I. Degr. 5th for *Major*, r. *Minor*. *P.* 162. *l.* 14, 15. *Dele.* between 6th. *Major* and 7th. *Minor*. P. 199. l. 11, r. *Anomalous Phænomena*.

The two *Schemes* engraved, are to be placed thus.

That of the 4 Scales, againſt *Pag.* 155.
For Tuning an *Organ*, againſt *Pag.* 181.

THE *Natural Grounds and Principles* OF HARMONY.

The INTRODUCTION.

HArmony consists of Causes, Natural and Artificial, as of Matter and Form. The Material Part of it, is Sound or Voice. The Formal Part is, The Disposition of Sound or Voice into Harmony; which requires, as a preparative Cause, skilfull Composition: and, as an immediate Efficient, Artfull Performance.

The former Part, viz. *The Matter, lies deep in Nature, and requires much Research into Natural Philosophy to unfold it; to find how Sounds are made, and*

how they are first fitted by Nature for Harmony, before they be disposed by Art. Both together make Harmony compleat.

Harmony, then, results from Practick Musick, and is made by the Natural and Artificial Agreement of different Sounds, (viz. Grave and Acute,) by which the Sence of Hearing is delighted.

This is properly in Symphony, i. e. Consent of more Voices in different Tones; but is found also in solitary Musick of one Voice, by the Observation and Expectation of the Ear, comparing the Habitudes of the following Notes to those which did precede.

Now the Theory in Natural Philosophy, of the Grounds and Reasons of this Agreement of Sounds, and consequent Delight and Pleasure of the Ear, (leaving the Management of these Sounds to the Masters of Harmonick Composure, and the skilfull Artists in Performance) is the Subject of this Discourse. The Design whereof (for the Sake and Service of all Lovers of Musick,

ſick, and particularly the Gentlemen of Their Majeſties Chapell Royal) is, to lay down theſe Principles as ſhort, and intelligible, as the Subject Matter will bear.

Where the firſt thing Neceſſary, is a Conſideration of ſomewhat of the Nature of Sound in General; and then, more particularly, of Harmonick Sounds, &c.

CHAP. I.

Of Sound in General.

IN General (to pass by what is not pertinent to this Design) Sence and Experience confirm these following Properties of Sound.

1. All Sound is made by Motion, *viz.* by Percussion with Collision of the Air.

2. That Sound may be propagated, and carried to Distance, it requires a *Medium* by which to pass.

3. This *Medium* (to our Purpose) is Air.

4. As far as Sound is propagated along the *Medium*; so far also the Motion passeth. For (if we may not say that the Motion and Sound are one and the same thing, yet at least) it is

neceſſarily conſequent, that if the Motion ceaſe, the Sound muſt alſo ceaſe.

5. Sound, where it meets with no Obſtacle, paſſeth in a Sphere of the *Medium*, greater or leſs, according to the Force and Greatneſs of the Sound: Of which Sphere the ſonorous Body is as the Centre.

6. Sound, ſo far as it reacheth, paſſeth the *Medium*, not in an Inſtant, but in a certain uniform Degree of Velocity, calculated by *Gaſſendus*, to be about the Rate of 276 Paces, in the ſpace of a ſecond Minute of an Hour. And where it meets with any Obſtacle, it is ſubject to the Laws of Reflexion, which is the Cauſe of Eccho's, Meliorations, and Augmentations of Sound.

7. Sound, *i. e.* the Motion of Sound, or ſounding Motion, is carried through the *Medium* or Sphere of Activity, with an *Impetus* or Force which ſhakes the free

free *Medium*, and ſtrikes and ſhakes every Obſtacle it meets with, more or leſs, according to the vehemency of the Sound, and Nature of the Obſtacle, and Nearneſs of it to the Centre, or ſonorous Body. Thus the impetuous Motions of the Sound of Thunder, or of a Cannon, ſhake all before it, even to the breaking of Glaſs Windows, *&c.*

8. The Parts of the ſounding Body are moved with a Motion of Trembling, or Vibration, as is evident in a Bell or Pipe, and moſt manifeſt in the ſtring of a muſical Inſtrument.

9. This Trembling, or Vibration, is either equal and uniform, or elſe unequal and irregular; and again, ſwifter or ſlower, according to the Conſtitution of the ſonorous Body, and Quality and Manner of Percuſſion; and from hence ariſe Differences of Sounds.

10. The Trembling, or Vibration

of the ſonorous Body, by which the particular Sound is conſtituted and diſcriminated, is impreſſed upon, and carried along the *Medium* in the ſame Figure and Meaſure, otherwiſe it would not be the ſame Sound, when it arrives at a more diſtant Ear, *i. e.* the Tremblings and Vibrations (which may be called Undulations) of the Air or *Medium*, are all along of the ſame Velocity and Figure, with thoſe of the ſonorous Body, by which they are cauſed.

The Differences of Sounds, as of one Voice from another, *&c.* (beſides the Difference of Tune, which is cauſed by the Difference of Vibrations) ariſe from the Conſtitution and Figure, and other Accidents of the ſonorous Body.

11. If the ſonorous Body be requiſitely conſtituted, *i. e.* of Parts ſolid, or tenſe, and regular, fit, being ſtruck, to receive and expreſs the Tremulous Mo-

Motions of Sound, equally and ſwiftly, then it will render a certain and even Harmonical Tone or Tune, received with Pleaſure, and judged and meaſured by the Ear: Otherwiſe it will produce an obtuſe or uneven Sound, not giving any certain or diſcernable Tune.

Now this Tune, or Tuneable ſound, φθόγγος, i. e. φωνῆς πλῆσις ἐμμελὴς ἐπὶ μίαν τάσιν, An agreeable Cadence of Voice, at one Pitch or Tenſion. This tuneable Sound (I ſay) as it is capable of other Tenſions towards Acuteneſs, or Gravity, *i. e.* the Tenſions greater or leſs, the Tune graver or more acute, *i. e.* lower or higher, is the firſt Matter or Element of Muſick. And this Harmonick Sound comes next to be conſidered.

CHAP.

CHAP. II.

Of Sound Harmonick.

THE first and great Principle upon which the Nature of Harmonical Sounds is to be found out and discovered, is this: That the Tune of a Note (to speak in our vulgar Phrase) is constituted by the Measure and Proportion of Vibrations of the sonorous Body; I mean, of the Velocity of those Vibrations in their Recourses.

For, the frequenter the Vibrations are, the more acute is the Tune; the slower and fewer they are in the same Space of Time, by so much the more grave is the Tune. So that any given Note of a Tune, is made by one certain Measure of Velocity of Vibrations; *viz.* Such a certain Number of Courses

Courſes and Recourſes. *e. g.* of a Chord or String, in ſuch a certain Space of Time, doth conſtitute ſuch a certain determinate Tune. And all ſuch Sounds as are Uniſons, or of the ſame Tune with that given Note, though made upon whatſoever different Bodies, (as String, Bell, Pipe, *Larynx*, &c.) are made with Vibrations or Tremblings of thoſe Bodies, all equal each to other. And whatſoever Tuneable Sound is more acute, is made with Vibrations more ſwift; and whatſoever is more grave, is made with more ſlow Vibrations: And this is univerſally agreed upon, as moſt evident to Experience, and will be more manifeſt through the whole Theory.

And, That the Continuance of the Sound in the ſame Tune, to the laſt, (as may be perceived in Wire-ſtrings, which being once ſtruck will hold their Sound long) depends upon the Equality of Time of the Vibrations, from the

the greateſt Range till they come to ceaſe: And this perfectly makes out the following Theory of Conſonancy, and Diſſonancy.

Some of the Ancient Greek Authors of Muſick, took notice of Vibrations: And that the ſwifter Vibrations cauſed acuter, and the ſlower, graver Tones. And that the Mixture, or not Mixture of Motions creating ſeveral Intervals of Tune, was the Reaſon of their being concord or diſcord. And likewiſe, they found out the ſeveral Lengths of a Monochord, proportioned to the ſeveral Intervals of Harmonick Sounds: But they did not make out the Equality of Meaſure of Time of the Vibrations laſt ſpoken of, neither could be prepared to anſwer ſuch Objections, as might be made againſt the Continuity of the ſameneſs of Tune, during the Continuance of the Sound of a String, or a Bell, after it is ſtruck. Neither did any of them offer any Reaſons for the

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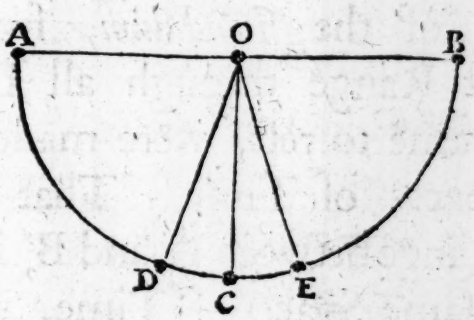
Proportions assigned, only it is said, that *Pythagoras* found them out by Chance.

But now, These (since the Acute *Galileo* hath observed, and discovered the Nature of *Pendulums*) are easie to be explained, which I shall do, premising some Consideration of the Properties of the Motions of a *Pendulum*.



Hang a Plumbet C on a String or Wire, fixed at O. Bear C to A: Then let it range freely, and it will move toward B, and from thence swing back towards A. The Motion from A to B, I call the Course, and back

back from B to A, the Recourſe of the *Pendulum*, making almoſt a Semi-Circle, of which O is the Centre. Then ſuffering the *Pendulum* to move of it ſelf forwards and backwards, the Range of it will at every Courſe and Recourſe abate, and diminiſh by degrees, till it come to reſt perpendicular at OC.

Now that which *Galileo* firſt obſerved, was, that all the Courſes and Recourſes of the *Pendulum*, from the greateſt Range through all Degrees till it came to reſt, were made in Equal Spaces of Time. That is, *e.g.* The Range between A and B, is made in the ſame Space of Time, with the Range between D and E, the Plumbet moving ſwifter between A and B, the greater Space; and ſlower between D and E, the leſſer; in ſuch Proportions, that the Motions between the Terms AB and DE, are performed in Equal ſpace of time.

And

And here it is to be noted, that where-ever in this Treatiſe, the ſwiftneſs or ſlowneſs of Vibrations is ſpoke of, it muſt be always underſtood of the Frequency of their Courſes and Recourſes, and not of the Motion by which it paſſeth from one ſide to another. For it is true, that the ſame *Pendulum* under the ſame Velocity of Returns, moves from one ſide to the other, with greater or leſs Velocity, according as the Range is greater or leſs.

And hence it is, that the Librations of a *Pendulum* are become ſo excellent, and uſefull a Meaſure of Time; eſpecially when a ſecond Obſervation is added, that, as you ſhorten the *Pendulum*, by bringing C nearer to its Centre O, ſo the Librations will be made proportionably in a ſhorter Meaſure of Time, and the Contrary if you lengthen it. And this is found to hold in a Duplicate Proportion of Length to Velocity.

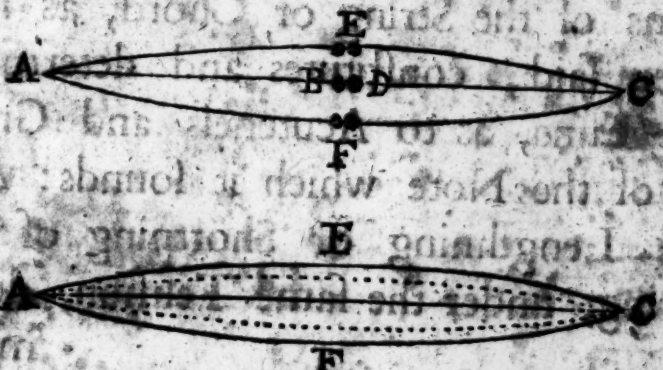
That

That is, the Length quadrupled, will ſubduple the Velocity of Vibrations: And the Length ſubquadrupled, will duple the Vibrations, for the Proportion holds reciprocally. As you add to the Length of the *Pendulum*, ſo you diminiſh the Frequency of Vibrations, and increaſe them by ſhortning it.

Now therefore to make the Courſes of a *Pendulum* doubly ſwift, *i. e.* to move twice in the ſame Space of Time, in which it did before move once; you muſt ſubquadruple the Length of it, *i. e.* make the *Pendulum* but a Quarter ſo long as it was before. And to make the Librations doubly ſlow, to paſs once in the Time they did paſs twice; you muſt quadruple the Length; make the *Pendulum* four Times as long as it was before, and ſo on in what Proportion you pleaſe.

Now to apply this to Muſick, make two *Pendulums*, AB and CD, faſten together the Plumbets B and D, and ſtretch

Stretch them at Length, (fixing the Centers A and C.) Then, being ſtruck, and put into Motion; the Vibrations, which before were Diſtinct, made by A B, and C D, will now be United (as of one Entire String) both backward and forward; between E and F. Which Vibrations (retaining the aforeſaid Analogy to a *Pendulum*) will be made in equal Spaces of Time, from the firſt to the laſt; *i. e.* from the greateſt Range to the leaſt, until they ceaſe. Now, this being a double *Pendulum*, to Subduple the Swiftneſs of the Vibrations, you do but Double the Length from A to C, which will be Quadruple to A B. The lower Figure is the ſame with that above, only the Plummets taken off.



C And

And here you have the Nature of the String of a Musical Instrument, resembling a double *Pendulum* moving upon two Centers, the Nut and the Bridge, and vibrating with the greatest Range in the Middle of its Length; and the Vibrations equal even to the last, which must make it keep the same Tune so long as it sounds. And because it doth manifestly keep the same Tune to the last, it follows that the Vibrations are equal; Confirming one another by two of our Senses: in that we see the Vibrations of a *Pendulum* move equally; and we hear the Tune of a String, when it is struck, continue the same.

The Measure of Swiftness of Vibrations of the String or Chord, as hath been said, constitutes and determines the Tune, as to Acuteness and Gravity of the Note which it sounds: And the Lengthning or Shortning of the String, under the same Tension, determines

mines the Meaſure of the Vibrations which it makes. And thus, Harmony comes under Mathematical Calculations of Proportions, of the Length of Chords; of the Meaſure of Time in Vibrations; of the Intervals of tuned Sounds. As the Length of one Chord to another, *Cæteris paribus*, I mean, being of the ſame Matter, Thickneſs and Tenſion; ſo is the Meaſure of the Time of their Vibrations. As the Time of Vibrations of one String to another, ſo is the Interval or Space of Acuteneſs or Gravity of the Tune of that one, to the Tune of the other: And conſequently, as the Length is (*Cæteris paribus*) ſo is the determinate Tune.

And upon theſe Proportions in the Differences, of Lengths, of Vibrations, and of Acuteneſs and Gravity; I ſhall inſiſt all along this Treatiſe, very largely and particularly, for the full Information of all ſuch ingenious Lovers of Muſick, as ſhall have the Curioſity to

inquire into the Natural Cauſes of Harmony, and of the *Phænomena* which occurr therein, though otherwiſe, to the more learned in Muſick and Mathematical Proportions, all might be expreſſed very much ſhorter, and ſtill be more ſhortned by the help of Symbols.

And here we may fix our Foot: Concluding, that what is evident to Sence, of theſe *Phænomena*, in a Chord, is equally (though not ſo diſcernably) true of the Motions of all other Bodies which render a tuneable Sound, as the Trembling of a Bell or Trumpet, the forming of the *Larynx* in our ſelves, and other Animals, the Throat of Pipes and of thoſe of an Organ, *&c.* All of them in ſeveral Proportions ſenſibly trembling and impreſſing the like Undulations of the *Medium*, as is done by the ſeveral (more manifeſt) Vibrations of Strings or Chords.

In theſe other Bodies, laſt ſpoken of,

we

we manifeſtly ſee Reaſon of the Difference of the Swiftneſs of their Vibrations (though we cannot ſo well meaſure them) from their Shape, and other Accidents in their Conſtitution; and chiefly from the Proportions of their Magnitudes; the Greater generally vibrating ſlower, and the Leſs more ſwiftly, which give the Tunes accordingly. We ſee it in the Greatneſs of a String. A greater and thicker Chord will give a graver and lower Tone than one that is more ſlender, of the ſame Tenſion and Length; but they may be made Uniſon by altering their Length and Tenſion.

Tenſion is proper to Chords or Strings (except you will account a Drumm for a Muſical Inſtrument, which hath a Tenſion not in Length, but in the whole Surface) as when we wind up, or let down the Strings, *i. e.* give them a greater or leſs Tenſion, in tuning a Viol, Lute, or Harpſichord,

and is of great Concern, and may be measured by hanging Weights on the String to give it Tension; but not easily, nor so certainly.

But the Lengths of Chords (because of their Analogy to a *Pendulum*) is chiefly considered, in the Discovery of the Proportions which belong to Harmony, it being most easie to measure and design the Parts of a Monochord, in relation to the whole String; and therefore all Intervals in Harmony may first be described, and understood, by the Proportions of the Length of Strings, and consequently of their Vibrations. And it is for that reason, that in this Treatise of the Grounds of Harmony, Chords come so much to be considered, rather than other sounding Bodies, & those, chiefly in their Proportions of Length. It is true, that in Wind-Instruments, there is a regard to the Length of Pipes, but they are not so well accommodated (as are Chords) to be exa-

examined, neither are their Vibrations, nor the Measure of them so manifest.

There are some Musical Sounds, which seem to be made, not by Vibrations but by Pulses, as by whisking swiftly over some Silk or Camblet-stuffs, or over the Teeth of a Comb, which render a kind of Tune more Acute or Grave, according to the Swiftness of the Motion. Here the Sound is made, not by Vibrations of the same Body, but by Percussion of several Equal, and Equidistant Bodies; as Threads of the Stuff, Teeth of the Comb, passing over them with the same Velocity as Vibrations are made. It gives the same Modification to the Tune, and to the Undulations of the Ayr, as is done by Vibrations of the same Measure; the Multiplicity of Pulses or Percussions, answering the Multiplicity of Vibrations. I take this notice of it, because others have done so; but I think it to be of no use in Musick.

APPENDIX.

Before I conclude this Chapter, it may seem needfull, better to confirm the Foundation we have laid, and give the Reader some more ample Satisfaction about the Motions and Measures of a Pendulum, *and the Application of it to Harmonick Motion.*

FIrst then, it is manifest to Sence and Experience, and out of all dispute; that the Courses and Recourses return sooner or later, *i.e.* more or less frequently, according as the *Pendulum* is shortned, or made longer. And that the Proportion by which the Frequency increaseth, is (at least) very near duplicate, *viz.* of the Length of the *Pendulum*, to the Number of Vibrations, but is in reverse, *i. e.* as the Length increaseth, so the Vibrations decrease; and contrary, quadruple the Length, and the Vibrations will be subdupled.

dupled. Subquadruple the Length, and the Vibrations will be dupled. And laſtly, that the Librations, the Courſes and Recourſes of the ſame *Pendulum*, are all made in equal Space of Time, or very near to it from the greateſt Range to the leaſt. Now though the duplicate Proportion aſſigned, and the Equality of Time, are a little called in queſtion, as not perfectly exact, though very near it; yet in a Monochord we find them perfectly agree, *viz.* as to the Length, Duple inſtead of Duplicate, becauſe a String faſtened at both Ends is as a double *Pendulum*, each of which is quadrupled by dupling the whole String. And on this duple Proportion, depend all the Rations found in Harmony. And again, the Vibrations of a String are exactly equal, becauſe they continue to give the ſame Tune.

Suppoſing then ſome little Difference may ſometime ſeem to be found

in

in either of theſe Motions of a *Pendulum*, yet the Nearneſs to Truth is enough to ſupport our Foundation, by ſhewing what is intended by Nature, though it ſometimes meet with ſecret Obſtacles in the *Pendulum*, which it does not in a well made String. We may juſtly make ſome Allowance for the Accidents, and unſeen Cauſes, which happen to make ſome little Variations in Trials of Motion upon groſs Matter, and conſequently the like for nicer Experiments made upon the *Pendulum*. It is difficult to find exactly the determinate Point of the Plumbet, which regulates the Motions of the *Pendulum*, and conſigns its juſt Length. Then obſerve the Varieties which happen through various Sorts of Matter, upon which Experiments are made. *Merſennus* tells us, that heavier Weights of the ſame Length move ſlower, ſo that whilſt a Lead Plumbet makes 39 Vibrations, Cork or Wood will make at

leaſt

leaſt 40. Again, that a ſtiff *Pendulum* vibrates more frequently, than that which hangs upon a Chord. So that a Barr of Iron, or Staff of Wood ought to be half as long again as the other, to make the Vibrations equal. Yet in each of theſe reſpectively to it ſelf, you will find the duplicate Proportion and Equality of Vibration, or as near as may be. And (as to Equality) though in the Extreams of the Ranges of Librations, *viz.* the Greateſt compared to the Leaſt, there may (from unſeen Cauſes) appear ſome Difference, yet there is do diſcernable Difference of the Time of Vibrations of a *Pendulum* in Ranges, that are near to one another, whether greater or leſs; which is the Caſe of the Ranges of the Vibration of a String being made in a very ſmall Compaſs: And therefore the Librations of a *Pendulum*, limited to a ſmall Difference of Ranges, do well correſpond with the Vibrations of a String.

As

As to Strings, the Whole of Harmony depends upon this experimented and unqueſtioned Truth, that Diapaſon is duple to its Uniſon, and conſequently Diapente is Seſquialterum, Diateſſeron Seſquitertium, &c. Yet if you happen to divide a faulty String of an Inſtrument, you will not find the Octave juſt in the Middle, nor the other Intervals in their due Proportion, which is no Default in Nature, but in the Matter we apply. A falſe String is that, which is thicker in one Part of its Length, than in another. The thicker Part naturally vibrates ſlower, and ſounds graver; the more ſlender Part vibrates ſwifter, and ſounds more acute. Thus whilſt two Sounds ſo near one another, are at once made upon the ſame String, they make a rough diſcording Jarr, being a hoarſe Tune mixed of both, more or leſs, as the String is more or leſs unequal: And if the thicker Part be next the Frets,

Frets, then the Fret (for example D. F. H. &c. in a Viol or Lute) will render the Tune of the Note too ſharp; and the contrary, if the ſlender Part of the String be next the Frets; becauſe in the former, the thicker Part is ſtopped, and the thinner ſounds more of the acuter Part of this unhappy Mixture: As in the latter, the thicker Part is left to ſound the graver Tune, and thus the Fret will give a wrong Tune, though the Fault be not in the Fret, but in the String; which yet, by an unwary Experimenter, may happen to cauſe the *Sectio Canonis* to be called in queſtion, as well as the Meaſures of a *Pendulum* are diſputed.

But all this does not diſprove the Meaſures found out, and aſſigned to Harmonick Intervals, which are verified upon a true String or Wire as to their Lengths, and as to the Equality of Recourſes in their Vibrations, though *Pendulums* are thought to move ſlower

in

in their leaſt Ranges; yet, as to Strings, in the very ſmall Ranges which they make, (which are much leſs in other Inſtruments, or ſounding Bodies) I need add no more than this, that the Continuance of the ſame Tune to the laſt, after a Chord is ſtruck, and the continued Motion in leſs Vibrations of a ſympathizing String, during the Continuance of greater Vibrations of the String which is ſtruck, do either of them ſufficiently demonſtrate, that thoſe greater or leſs Vibrations, are both made in the ſame Meaſure of Time, according to their Proportions, keeping exact Pace with each other. Otherwiſe; In the former, the Tune would ſenſibly alter, and in the latter, the ſympathizing String could not be continued in its Motion. This was not ſo well concluded, till the late Diſcoveries of the *Pendulum* gave light to it.

There is one thing more which I muſt

muſt not omit. That the Motions of a *Pendulum*, may ſeem not ſo proper to explicate the Motions of a String, becauſe the ſaid Motions depend upon differing Principles, *viz.* thoſe of a *Pendulum* upon Gravity; thoſe of a String upon Elaſticity. I ſhall therefore endeavour to ſhew, how the Motions of a *Pendulum*, agree with thoſe of a Spring, and how properly the Explication of the Vibrations of a String, is deduced from the Properties of a *Pendulum*.

The Elaſtick power of a Spring, in a Body indued with Elaſticity, ſeems to be nothing elſe, but a natural Propenſion and Endeavour of that Body, forced out of its own Place, or Poſture, to reſtore it ſelf again into its former more eaſie and natural Poſture of Reſt. And this is found in ſeveral Sorts of Bodies, and makes different Caſes, of which I ſhall mention ſome.

If the Violence be by Compreſſion, forcing

forcing a Body into leſs room than it naturally requires; then the Endeavour of Reſtitution, is by Dilatation to gain room enough. Thus Ayr may be compreſſed into a leſs Space, and then will have a great Elaſticity, and ſtruggle to regain its room. Thus, if you ſqueeze a dry Sponge, it will naturally, when you take off the Force, ſpread it ſelf, and fill its former Place. So, if you preſs with your Finger a blown Bladder, it will ſpring and riſe again to its Place. And to this may be reduced the Springs of a Watch, and of a Spiral Wire, *&c.*

Again, a ſtiff, but pliable Body, faſtened at one End, and drawn aſide at the other, will ſpring back to its former Place; this is the Caſe of Steel-ſprings of Locks, Snaſ-hances, *&c.* and Branches of Trees, when ſhaken with the Wind, or pulled aſide, return to their former Poſture: As is ſaid of the Palm, *Depreſſa Reſurgo.* And there are

are innumerable instances of this kind, where the Force is by bending, and the Restitution by unbending or returning.

This kind is resembled by a *Pendulum*, or Plummet hanging on a String, whose Gravity, like the Spring in those other Bodies, naturally carries it to its place, which here is downward; all heavy Bodies naturally descending till they meet with some Obstacle to rest upon. And the lowest that the Plummet can descend in its Restraint by the String, is, when it hangs perpendicular, as at A B; where it is nearest to the Horizontal Plane G H, and therefore lowest. Now, if you force the Plummet upward (held at length upon the String) from B to C, and let it go; it will, by a Spontaneous Motion, endeavour its Restitution to B: but, having nothing to stop it but Air, the Impulse of its own Velocity will carry it beyond B, towards D;

and ſo backward and forward, decreaſing at every Range, till it come to reſt at B.

Thus the *Pendulum* and Spring agree in Nature, if you conſider the Force againſt them, and their Endeavour of Reſtitution.

But further, if you take a thin ſtiff *Lamina* of Steel, like a Piece of Twopenny Riband of ſome length, and nail it faſt at one End, (the remainder of it being free in the Air) then force the other End aſide and let it go; it will make Vibrations backward and forward, perfectly anſwering thoſe of a *Pen-*

a *Pendulum*. And much more, if you contrive it with a little Steel Button at the End of it, both to help the Motion when once ſet on foot, and to bear it better againſt the Reſiſtance of the Ayr. There will be no difference between the Vibrations of this Spring, and of a *Pendulum*, which in both, will be alike increaſed or decreaſed in Proportion to their Lengths. The ſame End (*viz.* Reſt) being, in the ſame manner, obtained by Gravity in one, and Elaſticity in the other.

Further yet, if you nail the Spring above, and let it hang down perpendicular, with a heavier Weight at the lower End, and then ſet it on moving, the Vibrations will be continued and carried on both by Gravity and Elaſticity, the *Pendulum* and the Spring will be moſt friendly joyned to cauſe a ſimple equal Motion of Librations, I mean, an equal Meaſure of Time in the Recourſes; only the Spring an-

ſwerably to its Strength, may cauſe the Libra tions tobe ſomewhat ſwifter, as an Addition of Tenſion does to a String continued in the ſame length.

I come now to conſider a String of an Inſtrument, which is a Spring faſtned at both Ends. It acquireth a double Elaſticity. The firſt by Tenſion, and the Spring is ſtronger or weaker, according as the Tenſion is greater or leſs. And by how much ſtronger the Spring is, ſo much more frequent are the Vibrations, and by this Tenſion therefore, the Strings of an Inſtrument keeping the ſame length are put in Tune, and this Spring draws length-ways, endeavouring a Relaxation of the Tenſion.

But then, *Secondly*, the String being under a ſtated Tenſion, hath another Elaſtick Power ſide-ways, depending upon the former, by which it endeavours, if it be drawn aſide, to reſtord it ſelf to the eaſieſt Tenſion, in the ſhorteſt, *viz.* ſtreighteſt line. In

In the former Caſe, Tenſion doth the ſame with abatement of length, and affects the String properly as a Spring, in that the String being forcibly ſtretched, as forcibly draws back to regain the remiſs Poſture in which it was before: And bears little Analogy with the *Pendulum*, except in general, in their ſpontaneous Motions in order to their Reſtitution.

But there is great Correſpondence in the ſecond Caſe, between the Librations of a *Pendulum* and the Vibrations of a String (for ſo, for diſtinctions ſake, I will now call them) in that they are both proportioned to their length, as has been ſhewn; and between the Elaſticity which moves the String, and Gravity which moves the *Pendulum*, both of them having the ſame Tendency to Reſtitution, and by the ſame Method. As the Declivity of the Motion of a *Pendulum*, and conſequently the Impulſe of its Gravity is

ſtill leſſened in the Arch of its Range from a Semi-Circle, till it come to reſt perpendicular; the Deſcent thereof being more down-right at the firſt and greateſt Ranges, and more Horizontal at the laſt and ſhorteſt Ranges, as may be ſeen in the preceding Figure CI IE EB; ſo the Impulſe of a Spring is ſtill gradually leſſened as the Ranges ſhorten, and as it gains of relaxation, till it come to be reſtored to reſt in its ſhorteſt Line. And this may be the Cauſe of the Equality of Time of the Librations of a *Pendulum*, and alſo of the Vibrations of a String. Now, the Proportions of Length, to the Velocity of Vibrations in one, and of Librations in the other, we are ſure of, and find by manifeſt Experience to be quadruple in one, and duple in the other.

Now tack two equal *Pendulums* together (as before) being faſtened at both Ends, take away the Plumbets,

and

and you make it a String, retaining ſtill the ſame Properties of Motion, only what was ſaid before to be cauſed by Gravity, muſt now be ſaid to be done by Elaſticity. You ſee what an eaſie Step here is out of one into the other, and what Agreement there is between them. The *Phænomena* are the ſame, but difficultly experimented in a String, where the Vibrations are too ſwift to fall under exact Meaſure; but moſt eaſie in a *Pendulum*, whoſe ſlow Librations may be meaſured at pleaſure, and numbered by diſtant Moments of Time.

To bring it nearer, make your Tenſion of the String by Gravity, inſtead of ſcrewing it up with a Pegg or Pin: Hang weight upon a Pulley at one End of the String, and as you increaſe the Weight, ſo you do increaſe the Tenſion, and as you increaſe the Tenſion, ſo you increaſe the Velocity of Vibrations. So the Vibrations are

proportionably regulated immediately by Tenſion, and mediately by Gravity. So that Gravity may claim a ſhare in the Meaſures of theſe Harmonick Motions.

But to come ſtill nearer, and home to our purpoſe. Faſten a Gut or Wire-ſtring above, and hang a heavy Weight on it below, as the Weight is more or leſs, ſo will be the Tenſion, and conſequently the Vibrations. But let the ſame Weight continue, and the String will have a ſtated ſetled Tenſion. Here you have both in one, a *Pendulum*, and the Spring of a String, which reſembles a double *Pendulum*: Draw the Weight aſide, and let it range, and it is properly a *Pendulum*, librating after the Nature of a *Pendulum*. Again, when the Weight is at reſt, ſtrike the String with a gentle *Plectrum* made of a Quill, on the upper part, ſo as not to make the Weight move, and the String will vibrate, and

give its Tune, like other Strings fastned at both Ends, as this is also in this Case. So you have here both a *Pendulum* and a String, or either, which you please. And (the Tension being supposed to be setled under the same Weight) the common Measure and Regulator of the Proportions of them is the Length. And as you alter the Length, so proportionably you alter at once the Velocity in the Recourses of the Vibrations of the String, and of the Librations of the *Pendulum*. And though the Vibrations be so much swifter, and more frequent than the Librations, yet the Rations are altered alike. If you subduple the Length of the String, then the Vibrations will be dupled. And if you subquadruple it, then the Librations will be also dupled, allowing for so much of the Body of the Weight as must be taken in, to determine the Length of the *Pendulum*.

The

The Vibrations are altered in duple Proportion to the Librations, becauſe (as hath been ſhewn) the String is as a double *Pendulum*, either one of which ſuppoſed *Pendulums* is but half ſo long as the String, and is quadrupled by dupling the whole String. Still therefore the Proportion of their Alterations holds ſo certainly and regularly with the Proportion of every Change of their common Length, that, if you have the Comparative Ration of either of theſe two, *viz.* Vibrations or Librations to the Length, you have them both: The increaſe of the Velocity of Librations being ſubduple to the increaſe of the Velocity of Vibrations. And thus the Motions of a *Pendulum* do fully and properly diſcover to us, the Motions of a String, by the manifeſt Correſpondence of their Properties and Nature. The *Pendulum's* Motion of Gravity, and the Strings of Elaſticity bearing ſo certain Proportions

accord-

according to Length, that the Principles of Harmony, may be very properly made out, and moſt eaſily comprehended, as explained by the *Pendulum*. And we find, that in all Ages, this part of Harmony was never ſo clearly underſtood, as ſince the late Diſcoveries about the *Pendulum*.

And I chuſe to make this Illuſtration by the *Pendulum*, becauſe it is ſo eaſie for Experiment, and for our Comprehenſion; and the Elaſtick Power ſo difficult.

Having ſeen the Origine of Tuneable or Harmonick Sounds, and of their Difference in reſpect of Acuteneſs and Gravity: It is next to be conſidered, how they come to be affected with Conſonancy and Diſſonancy, and what theſe are.

CHAP.

CHAP. III.

Of Consonancy and Dissonancy.

COnsonancy and Dissonancy are the Result of the Agreement, mixture or uniting (or the contrary) of the undulated Motions of the Ayr or *Medium*, caused by the Vibrations by which the Sounds of distinct Tunes are made. And those are more or less capable of such Mixture or Co-incidence according to the Proportion of the Measures of Velocity in which they are made, *i.e.* according as they are more or less commensurate. This I might well set down as a *Postulatum*. But I shall by several Instances indeavour to illustrate the undulating Motions or Undulations of the Ayr; and confirm what is said of their Agreement and Disagreements. And first the

the Undulations, by ſomewhat we ſee in other Liquids.

Let a Stone drop into the Middle of a ſmall Pond of ſtanding Water when it is quiet, you ſhall ſee a Motion forthwith impreſſed upon the Water, paſſing and dilating from that Center where the Stone fell, in circular Waves one within another, ſtill propagated from the Center, ſpreading till they reach and daſh againſt the Banks, and then returning, if the force of the Motion be ſufficient, and meeting those inner Circles which purſue the ſame Courſe, without giving them any Check.

And if you drop a Stone in another place, from that Centre will likewiſe ſpread round Waves, which meeting the other, will quietly paſs them, each moving forwards in its own proper Figure.

The like is better experimented in Quick-ſilver, which being a more denſe

denſe Body, continues its Motions longer, and may be placed nearer your Eye. If you try it in a pretty large round Veſſel, ſuppoſe of a foot Diameter, the Waves will keep their own Motion forward and backward, and quietly paſs by one another as they meet. Something of this may be ſeen in a long narrow Paſſage, where there is not room to advance in Circles.

Make a wooden Trough or long Box, ſuppoſe of two Inches broad, and two deep, and twenty long. Fill it three Quarters or half full of Quick-ſilver, and place it Horizontally, when it is at quiet, give it with your Finger a little patt at one End, and it will impreſs a Motion of a ridgid Wave acroſs, which will paſs on to the other End, and daſhing againſt it, return in the ſame Manner, and daſh againſt the hether End, and go back again, and thus backward and forward, till

the

the Motion ceaſe. Now if after you have ſet this Motion on foot, you cauſe ſuch another, you ſhall ſee each Wave keep its regular Courſe; and when they meet one another, paſs on without any Reluctancy.

I do not ſay theſe Experiments are full to my purpoſe, becauſe theſe being upon ſingle Bodies, are not ſufficient to expreſs the Diſagreements of Diſproportionate Motions cauſed by different Vibrations of different ſounding Bodies; but theſe may ſerve to illuſtrate thoſe inviſible Undulations of Ayr. And how a Voice reflected by the Walls of a Room, or by Echo being of adequate Vibrations, returns from the Wall, and meets the commenſurate Undulations paſſing forwards, without hindring one another.

But there are Inſtances which further confirm the Reaſons of Conſonancy and Diſſonancy, by the Manifeſt

feſt agreeing or diſagreeing Meaſures of Motions already ſpoken of.

It hath been a common Practice to imitate a Tabour and Pipe upon an Organ. Sound together two diſcording Keys (the baſe Keys will ſhew it beſt, becauſe their Vibrations are ſlower) let them, for Example, be Ga-raut with Gamut ſharp, or F Faut ſharp, or all three together. Though theſe of themſelves ſhould be exceeding ſmooth and well voyced Pipes; yet, when ſtruck together, there will be ſuch a Battel in the Ayr between their diſproportioned Motions, ſuch a Chat-ter and Thumping, that it will be like the beating of a Drum, while a Jigg is played to it with the other hand. If you ceaſe this, and ſound a full Cloſe of Concords, it will appear ſurprizingly ſmooth and ſweet, which ſhews how Diſcords well placed, ſet off Concords in Compoſition. But I bring this Inſtance to ſhew, how ſtrong

and

and vehement these undulating Motions are, and how they correspond with the Vibrations by which they are made.

It may be worth the while, to relate an Experiment upon which I happened. Being in an Arched sounding Room near a shrill Bell of a House Clock, when the Alarm struck, I whistled to it, which I did with ease in the same Tune with the Bell, but, indeavouring to whistle a Note higher or lower, the Sound of the Bell and its cross Motions were so predominant, that my Breath and Lips were check'd so, that I could not whistle at all, nor make any Sound of it in that discording Tune. After, I sounded a shrill whistling Pipe, which was out of tune to the Bell, and their Motions so clashed, that they seemed to sound like switching one another in the Ayr.

Galileo, from this Doctrine of *Pendulums*, easily and naturally explains

the ſo much admired ſympathy of Conſonant ſtrings; one (though untouch'd) moving when the other is ſtruck. It is perceptible in Strings of the ſame, or another Inſtrument, by trembling ſo as to ſhake off a Straw laid upon the other String: But in the ſame Inſtrument, it may be made very viſible, as in a Baſs-viol. Strike one of the lower Strings with the Bow, hard and ſtrong, and if any of the other Strings be Uniſon or Octave to it, you ſhall plainly ſee it vibrate, and continue to doe ſo, as long as you continue the Stroke of your Bow, and, all the while, the other Strings which are diſſonant, reſt quiet.

The Reaſon hereof is this. When you ſtrike your String, the Progreſſive ſound of it ſtrikes and ſtarts all the other Strings, and every of them makes a Movement in its own proper Vibration. The Conſonant ſtring, keeping meaſure in its Vibrations with

thoſe

those of the sounding String hath its Motion continued, and propagated by continual agreeing Pulses or Strokes of the other. Whereas the Remainder of the Dissonant strings having no help, but being checked by the cross Motions of the sounding String, are constrained to remain still and quiet. Like as, if you stand before a *Pendulum*, and blow gently upon it as it passeth from you, and so again in its next Courses keeping exact time with it, it is most easily continued in its Motion. But if you blow irregularly in Measures different from the Measure of the Motion of the *Pendulum*, and so most frequently blow against it, the Motion of it will be so checked, that it must quickly cease.

And here we may take notice, (as hath been hinted before) that this also confirms the aforesaid Equality of the Time of Vibrations to the last, for that the small and weak Vibrations of

the ſympathizing String are regulated and continued by the Pulſes of the greater and ſtronger Vibrations of the ſounding String, which proves, that notwithſtanding that Diſparity of Range, they are commenſurate in the Time of their Motion.

This Experiment is ancient: I find it in *Ariſtides Quintilianus* a Greek Authour, who is ſuppoſed to have been contemporary with *Plutarch*. But the Reaſon of it deduced from the *Pendulum*, is new, and firſt diſcovered by *Galileo*.

It is an ordinary Trial, to find out the Tune of a Beer-glaſs without ſtriking it, by holding it near your Mouth, and humming loud to it, in ſeveral ſingle Tunes, and when you at laſt hitt upon the Tune of the Glaſs, it will tremble and Eccho to you. Which ſhews the Conſent and Uniformity of Vibrations of the ſame Tune, though in ſeveral Bodies.

To

To close this Chapter. I may conclude that Consonancy is the Passage of several Tuneable sounds through the *Medium*, frequently mixing and uniting in their undulated Motions, caused by the well proportioned commensurate Vibrations of the sonorous Bodies, and consequently arriving smooth, and sweet, and pleasant to the Ear. On the contrary, Dissonancy is from disproportionate Motions of Sounds, not mixing, but jarring and clashing as they pass, and arriving to the Ear Harsh, and Grating, and Offensive. And this, in the next Chapter shall be more amply explained.

Now, what Concords and Discords are thus produced, and in use, in order to Harmony, I shall next consider.

CHAP. IV.

Of Concords.

CConcords are Harmonic ſounds, which being joyned pleaſe and delight the Ear; and Diſcords the Contrary. So that it is indeed the Judgment of the Ear that determines which are Concords and which are Diſcords. And to that we muſt firſt reſort to find out their Number. And then we may after ſearch and examine how the natural Production of thoſe Sounds, diſpoſeth them to be pleaſing or unpleaſant. Like as the Palate is abſolute Judge of Taſts, what is ſweet, and what is bitter, or ſowr, *&c.* though there may be alſo found out ſome natural Cauſes of thoſe Qualities. But the Ear being entertained with Motions which fall under exact Demonſtrations of their Meaſures, the

Do-

Doctrine hereof is capable of being more accurately diſcovered.

Firſt then, (ſetting aſide the Uniſon, Concord, which is no Space or Interval, but an Identity of Tune) the Ear allows and approves theſe following Intervals, and only theſe for Concords to any given Note, *viz.* the Octave or Eighth, the Fifth, then the Fourth, (though by later Maſters of Muſick degraded from his Place) then the Third *Major*, the Third *Minor*, the Sixth *Major*, and the Sixth *Minor*. And alſo ſuch, as in the Compaſs of any Voice or Inſtrument beyond the Octave, may be compounded of theſe, for ſuch thoſe are, I mean compounded, and only the former Seven are ſimple Concords; not but that they may ſeem to be compounded, *viz.* the greater of the leſs within an Octave, and therefore may be called Syſtems, but they are Originals. Whereas beyond an Octave, all is but Repetition of theſe in Compound with

the Eighth, as a Tenth is an Eighth and a Third; a Twelfth is an Eighth and a Fifth; a Fifteenth is Diſdiapaſon, *i.e.* two Octaves, *&c.*

But notwithſtanding this Diſtinction of Original and Compound Concords; and, tho' theſe compounded Concords are found, and diſcerned by their Habitude to the Original Concords comprehended in the Syſtem of Diapaſon; (as a Tenth aſcending is an Octave above the Third, or a Third above the Octave; a Twelfth is an Octave to the Fifth, or a Fifth to the Eighth, a Fifteenth is an Eighth above the Octave, *i.e.* Diſdiapaſon two Eighths, *&c.*) yet they muſt be own'd, and are to be eſteemed good and true Concords, and equally uſefull in Melody, eſpecially in that of Conſort.

The Syſtem of an Eighth, containing ſeven Intervals, or Spaces, or Degrees, and eight Notes reckoned incluſively, as expreſſed by eight Chords,

is called Diapaſon, *i. e.* a Syſtem of all intermediate Concords, which were anciently reputed to be only the Fifth and the Fourth, and it comprehends them both, as being compounded of them both: And now, that the Thirds and Sixths are admitted for Concords, the Eighth contains them alſo: *Viz.* a Third *Major* and Sixth *Minor*, and again a Third *Minor* and Sixth *Major*. The Octave being but a Replication of the Uniſon, or given Note below it, and the ſame, as it were in Minuture, it cloſeth and terminates the firſt perfect Syſtem, and the next Octave above it aſcends by the ſame Intervals, and is in like manner compounded of them, and ſo on, as far as you can proceed upwards or downwards with Voices or Inſtruments, as may be ſeen in an Organ, or Harpſichord. It is therefore moſt juſtly judged by the Ear, to be the Chief of all Concords, and is the only Conſonant Syſtem, which being

being added to it ſelf, ſtill makes Concords.

And to it all other Concords agree, and are Conſonant, though they do not all agree to each other; nor any of them make a Concord if added to it ſelf, and the Complement or Reſidue of any Concord to Diapaſon, is alſo Concord.

The next in Dignity is the Fifth, then the Fourth, Third *Major*, Third *Minor*, Sixth *Major*, and laſtly Sixth *Minor*; all taken by Aſcent from the Uniſon or given Note.

By Uniſon is meant, ſometimes the Habitude or Ration of Equality of two Notes compared together, being of the very ſame Tune. Sometimes (as here) for the given ſingle Note to which the Diſtance, or the Rations of other Intervals are compared. As, if we conſider the Relations to *Gamut*, to which *A re* is a Tone or Second, *B mi* a Third, C a Fourth, D a Fifth, *&c.*

We

We call *Gamut* the Uniſon, for want of a more proper Word. Thus *C ſa ut*, or any other Note to which other Intervals are taken, may be called the Uniſon.

And the Reader may eaſily diſcern, in which Senſe it is taken all along by the Coherence of the Diſcourſe.

I come now to conſider the natural Reaſons, why Concords pleaſe the Ear, by examining the Motions by which all Concords are made, which having been generally alledged in the beginning of the third Chapter, ſhall now more particularly be diſcuſſed.

And here I hope the Reader will pardon ſome Repetition in a Subject, that ſtands in need of all Light that may be, if, for his eaſie and more ſteady Progreſs, before I proceed, I call him back to a Review and brief Summary of ſome of thoſe Notions, which have been premis'd and conſidered more at large. I have ſhewed,

1. That

1. That Harmonick Sound or Tune is made by equal Vibrations or Tremblings of a Body fitly conſtituted.

2. That thoſe Vibrations make their Courſes and Recourſes in the ſame Meaſure of Time; from the greateſt Range to the leſſer, till they come to reſt.

3. That thoſe Vibrations are under a certain Meaſure of Frequency of Courſes and Recourſes in a given Space of Time.

4. That if the Vibrations be more frequent, the Tune will be proportionably more Acute: if leſs frequent, more Grave.

5. That the Librations of a *Pendulum* become doubly frequent, if the *Pendulum* be made four times ſhorter; and twice ſlower, if the *Pendulum* be four times longer.

6. That a Chord, or String of a Muſical Inſtrument, is, as a double

Pendu-

Pendulum, or two *Pendulums* tacked together at length, and therefore hath the ſame Effects by dupling; as a *Pendulum* by quadrupling, *i. e.* by dupling the Length of the Chord, the Vibrations will be ſubdupled, *i. e.* be half ſo many in a given Time. And by ſubdupling the Length of the Chord, the Vibrations will be dupled, and proportionably ſo in all other Meaſures of Length, the Vibrations bearing a Reciprocal proportion to the Length.

7. That theſe Vibrations impreſs a Motion of Undulation or Trembling in the *Medium* (as far as the Motion extends) of the ſame Meaſure with the Vibrations.

8. That if the Motions made by different Chords be ſo commenſurate, that they mix and unite; bear the ſame Courſe either altogether, or alternately, or frequently: Then the Sounds of thoſe different Chords, thus mixing,

will

will calmly paſs the *Medium*, and arrive at the Ear as one Sound, or near the ſame, and ſo will ſmoothly and evenly ſtrike the Ear with Pleaſure, and this is Conſonancy, and from the want of ſuch Mixture is Diſſonancy. I may add, that as the more frequent Mixture or Coincidence of Vibrations, render the Concords generally ſo much the more perfect: So, the leſs there is of Mixture, the greater and more harſh will be the Diſcord.

From the Premiſſes, it will be eaſie to comprehend the natural Reaſon, why the Ear is delighted with thoſe forenamed Concords; and that is, becauſe they all unite in their Motions often, and at the leaſt at every ſixth Courſe of Vibration, which appears from the Rations by which they are conſtituted, which are all contained within that Number, and all Rations contained within that Space of Six, make Concords, becauſe the Mixture

of

of their Motions is answerable to the Ration of them, and are made at or before every Sixth Course. This will appear if we examine their Motions. First, how and why the Unisons agree so perfectly; and then finding the reason of an Octave, and fixing that, all the rest will follow.

To this purpose, strike a Chord of a sounding Instrument, and at the same Time, another Chord supposed to be in all respects Equal, *i. e.* in Length, Matter, Thickness, and Tension. Here then, both the Strings give their Sound; each Sound is a certain Tune; each Tune is made by a certain Measure of Vibrations: the same Vibrations are impressed upon, and carried every way along the *Medium*, in Undulations of the same Measure with them, until the Sounds arrive at the Ear. Now the Chords being supposed to be equal in all respects; it follows, that their Vibrations must be

also

alſo equal, and conſequently move in the ſame Meaſure, joyning and uniting in every Courſe and Recourſe, and keeping ſtill the ſame Equality, and Mixture of Motions of the String, and in the *Medium*. Therefore the Habitude of theſe two Strings is called Uniſon, and is ſo perfectly Conſonant, that it is an Identity of Tune, there being no Interval or Space between them. And the Ear can hardly judge, whether the Sound be made by two Strings, or by one.

But Conſonancy is more properly conſidered, as an Interval, or Space between Tones of different Acuteneſs or Gravity. And amongſt them, the moſt perfect is that which comes neareſt to Uniſon, (I do not mean betwixt which there is the leaſt Difference of Interval; but, in whoſe Motions there is the greateſt Mixture and Agreement next to Uniſon.) The Motions of two Uniſons are in Ration of 1 to 1, or

of

of Equality. The next Ration in whole Numbers is 2 to 1, Duple. Divide a Monochord in two Equal parts, half the Length compared to the whole, being in Subduple Ration, will make double Vibrations, making two Recourses in the same time that the other makes one, and so uniting and mixing alternately, *i. e.* every other Motion. Then comparing the Sounds of these two, and the half will be found to sound an Octave to the whole Chord. Now the Octave (ascending from the Unison) being thus found and fixed to be in duple Proportion of Vibrations, and subduple of Length; consequently the Proportions of all other Intervals are easily found out.

They are found out by resolving or dividing the Octave into the mean Rations which are contained in it. *Euclid*, in his *Sectio Canonis*, *Theorem.* 6. gives two Demonstrations to prove,

that Duple Ration contains, and is compoſed of the two next Rations, *viz.* *Seſquialtera* and *Seſquitertia*. Therefore an Octave which is in Duple Ration 2 to 1 is divided into, and compoſed of a Fifth, whoſe Ration is found to be *Seſquialtera* 3 to 2; and a Fourth, whoſe Ration is *Seſquitertia* 4 to 3. In like manner, *Seſquialtera* is compoſed of *Seſquiquarta* and *Seſquiquinta*. That is, a Fifth 3 to 2 may be divided into a Third *Major* 5 to 4, and a Third *Minor* 6 to 5; *&c.*

There is an eaſie way to take a view of the Mean Rations, which may be contained in any Ration given, by transferring the Prime or Radical Numbers of the given Ration into greater Numbers of the ſame Ration, as 2 to 1 into 4 to 2, or 6 to 3, *&c.* which have the ſame Ration of Duple. Again, 3 to 2 into 6 to 4, which is ſtill *Seſquialtera*. Now in 4 to 2, the Mediety is 3. So that

4 to

4 to 3 and 3 to 2 are comprehended in 4 to 2; that is, a Fourth and a Fifth are comprehended in an Eighth. In 6 to 4 the Mediety is 5, ſo 6 to 4 contains 6 to 5 and 5 to 4; *i. e.* a Fifth contains the 2 Thirds. Let 6 to 3 be the Octave, and it contains 6 to 5 Third *leſs*, 5 to 4 Third *Major*, and 4 to 3, a Fourth, and hath two Medieties, 5 and 4. Of this I ſhall ſay more in the next Chapter.

Theſe Rations expreſs the Difference of Length in ſeveral Strings which make the Concords; and conſequently the Difference of their Vibrations. Take two Strings A B, in all other Reſpects equal, and compare their Lengths, which if equal, make Uniſon or the ſame Tune. If A be double in Length to B, *i. e.* 2 to 1, the Vibrations of B will be duple to thoſe of A, and unite alternately, *viz.* at every Courſe, croſſing at the Recourſe, and give the Sound of an Octave to A.

If the Length of A be to that of B as 3 to 2, and consequently the Vibrations as 2 to 3, their Sounds will consort in a Fifth, and their Motions unite after every second Recourse, *i. e.* at every other or third Course.

If A to B, be as 4 to 3, they sound a Fourth, their Motions uniting after every third Recourse, *viz.* at every fourth Course.

If A to B, be as 5 to 4, they sound a Ditone, or third *Major*, and unite after every fourth Recourse, *i. e.* every fifth Recourse.

If A to B, be as 6 to 5, they sound a Trihemitone, or Third *Minor*, uniting after every fifth Recourse, at every sixth Course.

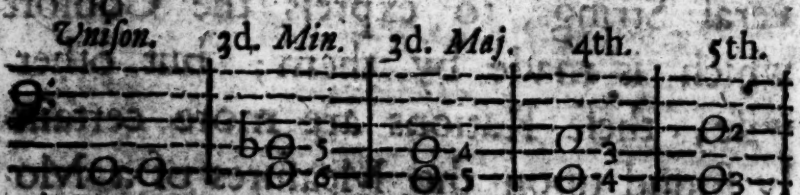
Thus by the frequency of their being mixed and united, the Harmony of joyned Concords is found so very sweet and pleasing; the Remoter being also combined by their Relation to other Concords besides the Unison.

The

The greater Sixth, 5 to 3, is within the Compass of Rations between 1 and 6; but, I confess, the lesser Sixth, 8 to 5, is beyond it: but is the Complement of 6 to 5 to an Octave, and makes a better Concord by its Combinations with the Octave, and Fourth from the Unison; having the Relation of a Third *Minor* to One, and of a Third *Major* to the Other, and their Motions uniting accordingly. And the Sixth *Major* hath the same Advantage. Of these Combinations I shall have occasion to say somewhat more, after I have made the Subject in hand as plain as I can.

I proposed the Collating of two several Strings, to express the Consort which is made by them; but otherwise, these Rations are more certainly found upon the Measures of a Monochord, taken, by being applied to the Section of a Canon, or a Rule of the Strings length divided into parts, as occasion requires: because there is no need

so often to repeat *Cæteris paribus*, as is when several Strings are collated. And if you take the Rations as Fractions, it will be more easie to measure out the given Parts of a Monochord, or single String extended on an Instrument: Those parts of the String divided by a Moveable Bridge or Fret put under, and made to sound; That Sound, related to the Sound of the Whole, will give the Interval sought after. *Ex. gr.* $\frac{1}{2}$ of the Chord gives an Eighth, $\frac{2}{3}$ give a Fifth, $\frac{3}{4}$ sound a Fourth, $\frac{4}{5}$ sound a Third *Major*, $\frac{5}{6}$ a Third *Minor*, $\frac{3}{5}$ a Sixth *Major*, $\frac{5}{8}$ a Sixth *Minor*: Now we thus express these Concords.



I ſaid, that all Concords are in Rations within the Number Six; and I may add, that all Rations within the Number Six, are Concords: Of which, take the following Scheme.

6	to 5	3d	*Minor.*	4	to 3	4th		6	to 5	3d	*Minor.*
	to 4	5th			to 2	8th		5	to 4	3d	*Major.*
	to 3	8th			to 1	15th		4	to 3	4th	
	to 2	12th						3	to 2	5th	
	to 1	19th		3	to 2	5th		2	to 1	8th	
					to 1	12th					
5	to 4	3d	*Major.*	2	to 1	8th					
	to 3	6th	*Major.*								
	to 2	10th	*Major.*								
	to 1	17th	*Major.*								

All that are Concords to the Uniſon, are alſo Concords to the Octave. And all that are Diſcords to the Uniſon, are Diſcords to the Octave. And ſome of the Intermediate Concords, are Concords one to another; as the two Thirds to the Fifth, and the Fourth to the two Sixths. So that the Uniſon, Third, Fifth, and Octave; or the Uniſon, Fourth, Sixth, and Octave, may

be ſounded together to make a compleat Cloſe of Harmony: I do not mean a Cloſe to Conclude with, for the Plagal is not ſuch; but a compleat Cloſe, as it includes all Concords within the Compaſs of Diapaſon. A Scheme of which I have ſet down at the End of the foregoing Staff of five Lines, which containeth the Notes by which the aforeſaid Concords are expreſſed. The former two which aſcend from the Uniſon, *Gamut*, by Third *Major* (or *Minor*) and Fifth, up to the Octave; are uſually called Authentick, as relating principally to the Uniſon, and beſt ſatisfying the Ear to reſt upon: The other two, which aſcend by the Fourth and Sixth *Minor*, (or *Major*) up to the ſame Octave, are called Plagal, as more combining with the Octave, ſeeming to require a more proper baſe Note, *viz.* an Eighth below the Fourth, and therefore not making a good concludig Cloſe: And on the continual

ſhifting

ſhifting theſe, or often changing them, depends the Variety of Harmony (as far as Conſonancy reacheth, which is but as the Body of Muſick) in all Contrapunct chiefly, but indeed in all Kinds of Compoſition. I do not exclude a Sprinkling of Diſcords; nor here medle with Ayr, Meaſure, and Rythmus, which are the Soul and Spirit of Muſick, and give it ſo great a commanding Power. The Plagal Moods deſcend by the ſame Intervals, by which the Authentick aſcend; which is by Thirds and Fifths; and the Authentick deſcend the ſame by which the Plagal aſcend, *viz.* by Fourths and Sixths; one chiefly relating to the Uniſon, the other to the Octave.

But that, for which I deſcribed theſe full Cloſes, was chiefly, to give (as I promis'd) a larger account of the before-mentioned Combinations of Concords, which increaſe the Conſonancies of each Note, and make a wonderfull

derfull Variegation and Delightfulneſs of the Harmony.

Caſt your Eye upon the Firſt of them in the Authentick Scale; you will ſee that *B mi* hath 3 Relations of Conſonnacy, *viz.* To the Uniſon, or given Note G; tothe Fifth, and to the Octave: To the Uniſon as a Third *Minor*; to the Fifth as a Third *Major*; to the Octave a Sixth *Major*; ſo that its Motions joyn after every fifth Recourſe, *i. e.* at every ſixth Courſe, with the Uniſon; every fifth with the Diapente or Fifth; every ſixth Courſe with the Octave. Then conſider the Diapente, *D ſol re*; as a Fifth to the Uniſon, it joyns with it every third Courſe; and as a Fourth to the Octave, they joyn every Fourth Courſe. Then, the Octave with the Uniſon, joyns after every ſecond Vibration, *i. e.* at every Courſe.

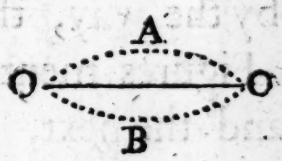
Now take a Review of the Variety of Conſonancies in theſe four Notes. Here are mixed together in one Con-

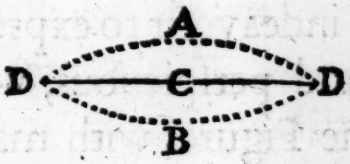
ſort

fort the Ratios of 2 to 1, 3 to 2, 4 to 3, 5 to 4, 6 to 5, 5 to 3. And just so it is in the other Closes, only changing alternately the Sixths. You may see here, within the Space of three Intervals from the Union, *viz.* 3d, 5th, and 8th; what a Concourse there is of Consonant Ratios to Variegate and give (as it were) a pleasant Purling to the Harmony within that Space. For now, all this Variety is formed within one System of Diapason, justly bearing that Name. But then, think what it will be, when the remote Compounded Concords are joyned to them; as when we make a full Close with both Hands upon an Organ, or Harpsichord, or when the higher Part of a Consort of Musick is reconciled to the lower, by the middle Parts; *viz.* the Treble to the Base, by the Mean and Tenor: And all this, refreshed by the Interchangings made between the Plagal and Authentick

tick Moods. Add to all this, the Infinite Variety of Movement of some Parts, through all Spaces, while some Part moves slowly: And (as in *Fuges*) one part chasing and pursuing another.

The whole Reason of Consonancy, being founded upon the Mixture, and Uniting of the Vibrating Motions of several Chords or sounding Bodies; it is fit, it should here be better explained and confirm'd. That their Mixtures accord to their Rations, it is easie to be computed: But it may be represented to your Eye.





V	A	B	B	A	A	B	B	A	A	B, &c.
O	AB	BA	AB	BA	AB	BA	AB	BA	AB	BA.

V	AB	BA	AB	BA	AB, &c.
D	ABC	CAB	BAC	CBA	ABC.

Let

Let V V be a Chord, and ſtand for the Uniſon: Let O O be a Chord half ſo long, which will be an Octave to the Uniſon, and the Vibrations double: Then I ſay, they will alternately, *i. e.* at every other Vibration unite: Let from A to B, be the Courſe of the Vibration, and from B to A the Recourſe. Obſerving by the way, that (in relation to the Figures mentioned in this Paragraph and the next, as alſo in the former Diagram of the *Pendulum*, Cap. 2. *pag.* 9.) When I ſay, [from B to A] and, [overtakes V, in A, *&c.*] I do there indeavour to expreſs the matter brief and perſpicuous, without perplexing the Figures with many Lines; and avoiding the Incumbrance of ſo many Cautions, whereby to diſtract the Reader: Yet I muſt always be underſtood to acknowledge the continual Decreaſe of the Range of Vibrations between A and B, while the Motion continues;

tinues; and by A and B, mean only the Extremities of the Range of all those Vibrations, both the First greateſt, and alſo the Succeſſive leſſened, and gradually contracted Extremities of their Range. And the following Demonſtration proceeds and holds equally in both, being applied to the Velocity of Recourſes, and not to the Compaſs of their Range, which is not at all here conſidered. Such a kind of Equity, I muſt ſometimes in other parts of this Diſcourſe, beg of the Candid Reader. To proceed therefore, I ſay, whilſt V being ſtruck, makes his Courſe from A to B; O (ſtruck likewiſe) will have his Courſe from A to B, and Recourſe from B to A. Next, whilſt V makes Recourſe from B to A; O is making its Courſe contrary, from A to B, but recourſeth and overtakes V in A, and then they are united in A, and begin their Courſe together. So you ſee, that the Vibrations of Diapaſon unite alter-

alternately, joyning at every Courſe of the Uniſon, and croſſing at the Recourſe.

Thus alſo Diapente or Fifth having the Ration of 3 to 2, unites in like manner at every third Courſe of the Uniſon. Let the Chord D D be Diapente to the Uniſon V; whilſt V courſeth from A to B, the Chord D courſeth from A to B, and makes half his Recourſe as far as C; *i. e.* 3 to 2. Whilſt V recourſeth from B to A, D paſſeth from C to A, and back from A to B. Whilſt V courſeth again from A to B, D paſſeth from B to A, and back to C. Whilſt V recourſeth from B to A, D paſſeth from C to B, and back to A: And then they unite in A, beginning their Courſes together, at every third Courſe of V. In like manner the reſt of the Concords unite, at the 4th, 5th, 6th Courſe, according to their Rations, as might this ſame way be ſhewn; but it would take

up

up too much room, and is needleſs; being made evident enough from theſe Examples already given.

Thus far the Rates and Meaſures of Conſonance lead us on, and give us the true and demonſtrable grounds of Harmony: But ſtill it is not compleat without Diſcords and Degrees (of which I ſhall treat in another Chapter) intermixed with the Concords, to give them a Foyl, and ſet them off the better. For, (to uſe a homely reſemblance) That our Food, taken alone, though proper, and wholſome, and natural, may not cloy the Palate, and abate the Appetite; the Cook finds ſuch kinds and varieties of Sawce, as quicken and pleaſe the Palate, and ſharpen the Appetite, though not feed the Stomach: As Vinegar, Muſtard, Pepper, &c. which nouriſh not, nor are taken alone, but carry down the Nouriſhment with better Reliſh, and aſſiſt it in Digeſtion. So the Practical

Maſters and Skilful Compoſers make uſe of Diſcords, judiciouſly taken, to reliſh the Conſort, and make the Concords arrive much ſweeter at the Ear, in all ſorts of Deſcant; but moſt frequently in Cadence to a Cloſe. In all which, the chief regard is to be had to what the Ear may expect in the Conduct of the Compoſition, and muſt be performed with Moderation and Judgment; which I now only mention, not intending to treat of Compoſing, which is out of my Deſign and Sphear, and would be too large; but my deſign is, to make theſe Grounds as plain as I can, thereby to gratifie thoſe, whoſe Philoſophical Learning, without previous Skill in Muſick, will eaſily render them capable of this Theory: And alſo, thoſe Maſters in Practick Muſick, and Lovers of it, who, though wanting Philoſophy, and the Latin and other Forreign Tongues, to read better Authors;

thors; yet, by the help of their knowledge in Muſick, may attain to underſtand the depth of the Grounds and Reaſons of Harmony, for whoſe ſakes it is done in this Language.

I ſhall conclude this Chapter with ſome Remarks, concerning the Names given to the ſeveral Concords: We call them *Third*, *Fourth*, *Fifth*, *Sixth*, and *Eighth*. Of theſe, the *Third's* being Two, and *Sixth's* being alſo Two, want better diſtinguiſhing Names. To call them Flat and Sharp Thirds, and Flat and Sharp Sixths is not enough, and lies under a miſtake; I mean, it is not a ſufficient Diſtinction, to call the greater Third and Sixth, Sharp Third, and Sharp Sixth; and the leſſer, Flat. They are ſo, indeed, in aſcending from the Uniſon; but in deſcending they are contrary; for to the Octave, that greater Sixth is a leſſer Third, and the greater Third is a leſſer Sixth; which leſſer Third and Sixth

cannot well be called Flat, being in a Sharp Key; Flat and Sharp therefore do not well distinguish them in General. The lesser Third from the Octave being sharp, and the greater Sixth flat. So, from the Fifth descending by Thirds, if the First be a *Minor* Third, it is Sharp, and the other being a *Major* Third, cannot be said to be Flat.

The other Distinction of them, *viz.* by *Major* and *Minor*, is more proper, and does well express which of them we mean. But still, the common and confused name of *Third*, if the Distinction of *Major* and *Minor* be not always well remembred, is apt to draw young Practitioners, who do not well consider, into another Errour. I would therefore call the greater Third (as the *Greeks* do) *Ditone*, *i. e.* of two whole Tones; and the Third *Minor*, *Trihemitone*, or *Sesquitone*, as consisting of three half Tones, (or rather,

ther of a Tone and half a Tone) And this would avoid the mentioned Errour which I am going to describe.

It is a Rule in composing Consort Musick, that it is not lawful to make a Movement of two Unisons, or two Eights, or two Fifths together; nor of two Fourths, unless made good by the addition of Thirds in another Part: But we may move as many Thirds or Sixths together as we please. Which last is false, if we keep to the same sort of Thirds and Sixths; for the two Thirds differ one from another in like manner as the Fourth differs from the Fifth. For in the same manner as the Eighth is divided into a Fifth and Fourth: so is a Fifth into a 3d *Major* and 3d *Minor*. Now call them by their right names, and, I say, it is not lawful to make a Movement of as many *Ditones*, or of as many *Sesquitones* as you please; and therefore when you take the liberty spoken

of, under the general names of Thirds, it will be found, that you mix *Ditones* and *Trihemitones*, and ſo are not concerned in the aforeſaid Rule; and ſo the Movements of Sixths will be made with mixture and interchanges of 6th *Major* and 6th *Minor*, which is ſafe enough.

Yet, I confeſs, there is a little more liberty in moving *Trihemitones* and *Ditones*, as likewiſe, either of the Sixths, than there is in moving Fourths or Fifths; and the Ear will bear it better. Nay, there is neceſſity, in a gradual Movement of Thirds, to make one Movement by two *Trihemitones* together in every Fourth, and Fifth, or Fourth disjunct: That is, twice in *Diapaſon*, or, at leaſt, in two Fifths; as in *Gamut* Key proper. The natural Aſcent will be, *Ut Re Mi Fa Sol La*: Now, to theſe join Thirds in Natural Aſcent, and they will be, *Mi Fa Sol La Fa Sol.* { *Mi Fa Sol La Fa Sol* / *Ut Re Mi Fa Sol La* } And thus it will be

be in other Cliffs, but with ſome variation, according to the place of the *Hemitone*. Here {Fa Re} and {Sol Mi} are two *Trihemitones* ſucceeding one another, and you cannot well alter them without diſordering the Aſcent, and diſturbing the Harmony; becauſe, where there is a *Hemitone*, the Tone below joined to it, makes a *Trihemitone*, and the next Tone above it, joined to it, makes the ſame. Thus you ſee the neceſſity of moving two *Trihemitones* together, twice in *Diapaſon*, or a 9th, in progreſſion of Thirds, in *Diatonic* Harmony, but you cannot well go further.

Now, there is Reaſon, why two *Trihemitones* will better bear it, becauſe of their different Relations, by which one *Trihemitone* is better diſtinguiſhed from another, than one Octave, or one Fifth, or one Fourth from another.

In a third *Minor*, which hath two Degrees or Intervals, conſiſting of a Tone and *Hemitone*, the *Hemitone* may be placed either in the lower Space, and then generally is united to his 3d *Major* (which makes the Complement of it to a Fifth) downward, and makes a ſharp Key; or elſe it may be placed in the upper Space, and then generally takes his 3d *Major* above, to make up the 5th upward, and conſtitute a Flat Key. And thus a *Tritone* is avoided both ways. I ſay, if the *Hemitone*, in the 3d *Minor* be below, then the 3d *Major* lies below it, and the Air is ſharp. If the *Hemitone* be above, then the 3d *Major* lies above, and the Air is Flat. And thus the two *Minor* Thirds joined in conſequence of Movement, are differenced in their Relations, conſequent to the place of the *Hemitone*; which variety takes off all Nauſeouſneſs from the Movement, and renders it ſweet and pleaſant.

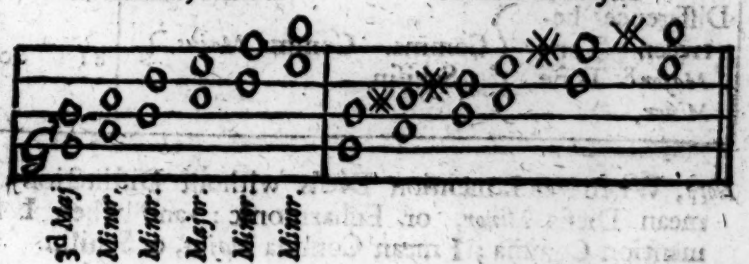
You

You cannot ſo well and regularly make a Movement of *Ditones*, though it may be done ſometimes, once or twice, or more, in a Bearing Paſſage (in like manner as you may ſometimes uſe Diſcords) to give, after a little grating, a better Reliſh. The Skilful Artiſt may go farther in the uſe of Thirds and Diſcords, than is ordinarily allowed.

I might enlarge this Chapter, by ſetting down Examples of the Lawful and Unlawful Movements of Thirds *Major* and *Minor*, and of the Uſe of Diſcords; but, as I ſaid before, my deſign is not to treat of Compoſition: However you may caſt your Eye upon theſe following Inſtances; and your own Obſervation from the beſt Maſters will furniſh you with the reſt.

Lawful Movement of Thirds, Mix'd.

Unlawful Movement of Thirds *Major*.



That

That the Reader may not incurr any Miſtake or Confuſion, by ſeveral Names of the ſame Intervals, I have here ſet them down together, with their Rations.

8th.	Octave. Diapaſon.		2	1
7th. *Major.*	Heptachord *Major.*		15	8
7th. *Minor.*	Heptachord *Minor.*		9	5
6th. *Major.*	Hexachord *Major.*		5	3
6th. *Minor.*	Hexachord *Minor.*		8	5
5th.	Diapente. Pentachord.		3	2
5th. *Falſe* (in defect.)	Semidiapente.		64	45
4th. *Falſe* (in exceſs.)	Tritone.		45	32
4th.	Diateſſaron. Tetrachord.		4	3
3d. *Major.*	Ditone.		5	4
3d. *Minor.*	Sexquitone. / Triemitone. / Semiditone.		6	5
2d. *Maj.* or Whole note *Major.*	Tone *Major.*	Degrees Diatonic. *Maxi.*	9	8
2d. *Min.* or Whole note *Minor.*	Tone *Minor.*	*Mini.*	10	9
2d. *Leſſ.* or Half-note *Greater.*	Hemi- / Semi- tone *Maj.*	*Minim.*	16	15
Half note *Leſſ.*	Hemi- / Semi- tone *Minor.* / Dieſis Chromatic. / Dieſis *Major.*		25	24
Quarter Note.	Dieſis Enharmonic. / Dieſis *Minor.*		128	125
Difference between Tone *Major* & Tone *Minor.*	Comma. Comma *Majus.* / Schiſm.		81	80

Note, Whenever I mention Dieſis without Diſtinction, I mean Dieſis *Minor*, or Enharmonic: and when I ſo mention Comma; I mean Comma *Majus*, or Schiſm.

I should next treat of Discords, but because there will intervene so much use of Calculation, it is needful that (before I gø further) I premise some account of Proportion in General, and apply it to Harmony.

CHAP.

CHAP. V.

Of Proportion; and Applyed to Harmony.

WHereas it hath been ſaid before, That Harmonick Bodies and Motions fall under Numerical Calculations, and the Rations of Concords have been already aſſign'd: It may ſeem neceſſary here (before we proceed to ſpeak of Diſcords) to ſhew the manner how to calculate the Proportions appertaining to Harmonick Sounds: And for this, I ſhall better prepare the Reader, by premiſing ſomething concerning Proportion in General.

We may compare (*i. e.* amongſt themſelves) either (1.) *Magnitudes*, (ſo they be of the ſame kind;) Or (2.) the *Gravitations*, *Motions*, *Velocities*, *Durations*,

Durations, *Sounds*, &c. from thence arising; or further, if you please, the Numbers themselves, by which the things Compared, are Explicated. And if these shall be Unequal, we may then consider, either, *First*, How much one of them Exceeds the other; or *Secondly*, After what manner one of them stands related to the other, as to the Quotient of the Antecedent (or former Term) divided by the Consequent (or latter Term:) Which Quotient doth Expound, Denominate, or shew, how many times, or how much of a time, or times, one of them doth contain the other. And this by the *Greeks* is called λόγος, *Ratio*; as they are wont to call the *Similitude*, or *Equality* of Ratio's, ἀναλογία, *Analogie*, *Proportion*, or *Proportionality*. But Custome, and the Sense assisting, will render any over-curious Application of these Terms unnecessary.

From

From these two Considerations last mention'd, there are wont to be deduced three sorts of Proportion, *Arithmetical*, *Geometrical*, and a mixt Proportion, resulting from these two, called *Harmonical*.

1. *Arithmetical*, When three or more Numbers in Progression, have the same Difference; as, 2, 4, 6, 8, &c. or discontinued, as 2, 4, 6; 14, 16, 18.

2. *Geometrical*, When three or more Numbers have the same Ration; as 2, 4, 8, 16, 32; or Discontinued; 2, 4; 64, 128.

Lastly, *Harmonical*, (partaking of both the other) When three Numbers are so ordered, that there be the same Ration of the Greatest to the Least; as there is of the Difference of the two Greater, to the Difference of the two less Numbers. As in these three Terms; 3, 4, 6; the Ration of 6 to 3 (being the greatest and least Terms)

Terms) is Duple. So is 2, the Difference of 6 and 4 (the two greater Numbers) to 1. the Difference of 4 and 3 (the two leſs Numbers) Duple alſo. This is Proportion Harmonical, which Diapaſon 6 to 3, bears to Diapente 6 to 4, and Diateſſaron 4 to 3; as its mean Proportionals.

Now for the kinds of Rations moſt properly ſo called; *i. e. Geometrical*; firſt obſerve, that in all Rations, the former Term or Number (whether greater or leſs) is always called the Antecedent; and the other following Number, is called the Conſequent. If therefore the Antecedent be the greater Term; then the Ration is either *Multiplex*, *Superparticular*, *Superpartient*, or (what is compounded of theſe) *Multiplex Superparticular*, or *Multiplex Superpartient*.

1. *Multiplex*; as Duple, 4 to 2; Triple, 6 to 2; Quadruple, 8 to 2.

2. *Super-*

2. *Superparticular*; as 3 to 2, 4 to 3, 5 to 4; Exceeding but by one aliquot part, and in their Radical, or leaſt Numbers, always but by one; and theſe Rations are termed *Seſquialtera*, *Seſquitertia*, (or *Supertertia*) *Seſquiquarta* (or *Superquarta*) *&c.* Note, that Numbers exceeding more than by one, and but by one aliquot part, may yet be *Superparticular*, if they be not expreſſed in their Radical, *i. e.* leaſt Numbers; as 12 to 8 hath the ſame Ration as 3 to 2; *i. e. Superparticular*; though it ſeem not ſo, till it be reduced by the greateſt Common Diviſor to its Radical Numbers 3 to 2. And the Common Diviſor (*i. e.* the Number by which both the Terms may ſeverally be divided) is often the Difference between the two Numbers; as in 12 to 8, the Difference is 4, which is the Common Diviſor. Divide 12 by 4, the Quotient is 3; Divide 8 by 4, the Quotient 2; ſo the Radical

is

is 3 to 2. Thus alſo 15 to 10, divided by the difference 5, gives 3 to 2; yet in 16 to 10, 2 is the common Diviſor, and gives 8 to 5; being *Superpartient*. But in all *Superparticular* Rations, whoſe Terms are thus made larger by being Multiplied: the Difference between the Terms is always the greateſt common Diviſor; as in the foregoing Examples.

The Third kind of Ration, is *Superpartient*, exceeding by more than One; as 5 to 3, which is called, *Superbipartiens Tertias* (or *Tria*) containing 3 and $\frac{2}{3}$; 8 to 5, *Supertripartiens Quintas*, 5 and $\frac{3}{5}$.

The Fourth is *Multiplex Superparticular*, as 9 to 4, which is Duple, and *Seſquiquarta*; 13 to 4, which is Triple, and *Seſquiquarta*.

The Fifth and laſt is *Multiplex Superpartient*, as 11 to 4; Duple, and *Supertripartiens Quartas*.

When the Antecedent is leſs than the Conſequent; *viz.* when a leſs is compared to a greater; then the ſame Terms ſerve to expreſs the Rations, only prefixing *Sub* to them, as *Submultiplex*, *Subſuperparticular* (or *Subparticular*) *Subſuperpartient* (or *Subpartient*) *&c.* 4 to 2 is *Duple*: 2 to 4 is *Subduple*. 4 to 3 is *Seſquitertia*; 3 to 4 is *Subſeſquitertia*; 5 to 3 is *Superbipartiens Tertias*; 3 to 5 is *Subſuperbipartiens Tertias*, *&c.*

This ſhort account of Proportion was neceſſary, becauſe almoſt all the Philoſophy of Harmony conſiſts in Rations, Of the Bodies; Of the Motions; and of the Intervals of Sound; by which Harmony is made.

And in ſearching, ſtating, and comparing the Rations of theſe, there is found ſo much Variety, and Certainty, and Facility of Calculation, that the Contemplation of them may ſeem not much leſs delightful, than the

the very hearing the good Muſick it ſelf, which ſprings from this Fountain. And thoſe who have already an affection for Muſick, cannot but find it improv'd and much inhanſed by this pleaſant recreating Chaſe (as I may call it) in the Large Field of Harmonic Rations and Proportions. where they will find, to their great Pleaſure and Satisfaction, the hidden cauſes of Harmony (hidden to moſt, even to Practitioners themſelves) ſo amply diſcovered and laid plain before them.

All the Habitudes of Rations to each other, are found by Multiplication or Diviſion of their Terms; by which any Ration is Added to, or Subſtracted from another. And there may be uſe of Progreſſion of Rations; or Proportions; and of finding a *Medium*, or *Mediety* between the Terms of any Ration. But the main work is done by Addition, and Subſtraction

ſtraction of Rations; which, though they are not performed like Addition and Subſtraction of Simple Numbers in Arithmetick; but upon Algebraic Grounds; yet the Praxis is moſt eaſie.

One Ration is added to another Ration, by Multiplying the two Antecedent Terms together, *i. e.* the Antecedent of one of the Rations, by the Antecedent of the other (for the more eaſe, they ſhould be reduc'd into their leaſt Numbers or Terms) And then the two Conſequent Terms in like manner. The Ration of the Product of the Antecedents, to that of the Product of the Conſequents, is equal to the other Two added or joined together. Thus (for Example) Add the Ration of 8 to 6; *i. e.* (in Radical Numbers) 4 to 3, to the Ration of 12 to 10; *i. e.* 6. to 5; the Product will be 24 and 15; *i. e.* 8 to 5; You may ſet them thus, and multiply 4 by 6, they

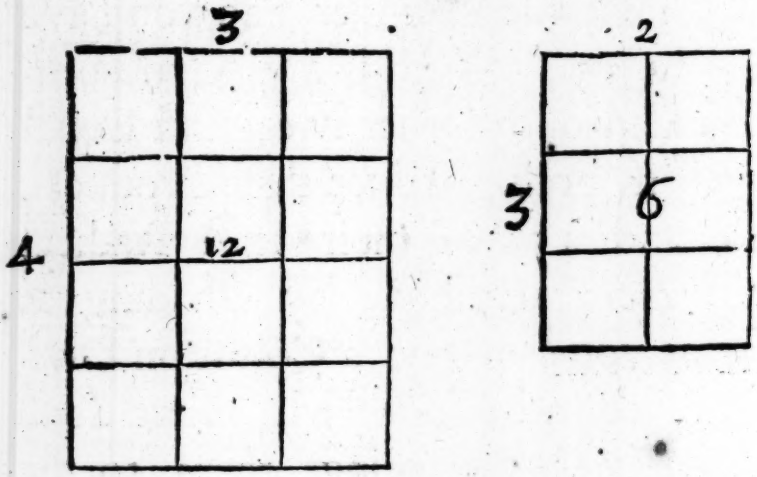
4	3
6	5
24.	15.

make

make 24, which ſet at the Bottom; then multiply 3 by 5, they make 15; which likewiſe ſet under, and you have 24 to 15; which is a Ration compounded of the other two, and Equal to them both. Reduce theſe Products, 24 and 15, to their leaſt Radical Numbers, which is, by dividing as far as you can find a Common Diviſor to them both (which is here done by 3) and that brings them to the Ration of 8 to 5. By this you ſee, that a Third *Minor*, 6 to 5; added to a Fourth, 4 to 3; makes a Sixth *Minor*, 8 to 5. If more Rations are to be added, ſet them all under each other, and multiply the firſt Antecedent by the Second, and that Product by the Third; and again, that Product by the Fourth, and ſo on; and in like manner the Conſequents.

This Operation depends upon the Fifth Propoſition of the Eighth Book of *Euclid*; where He ſhews, That the

Ration of plain Numbers is compounded of their ſides. See theſe Diagrams.



Now compound theſe Sides. Take for the Antecedents, 4 the greater Side of the greater Plane, and 3 the greater Side of the leſs Plane, and they multiply'd give 12; then take the remaining two Numbers 3 and 2, being the leſs Sides of the Planes (for Conſequents) and they give 6. So, the Sides of 4 and 3, and of 3 and 2 compounded (by multiplying the Antecedent Terms by themſelves, and the Conſequents by themſelves) make 12 to 6; *i. e.* 2 to 1. Which being apply'd,

apply'd, amounts to this; *Ratio Sesquialtera*, 3 to 2, added to *Ratio Sesquitertia* 4 to 3; makes Duple Ration, 2 to 1. Therefore *Diapente* added to *Diatessaron*, makes *Diapason*.

Substraction of One Ration from another greater, is performed in like manner, by Multiplying the Terms; but this is done not *Laterally*, as in Addition, but *Crosswise*; by Multiplying the Antecedent of the Former (*i. e.* of the Greater) by the Consequent of the Latter, which produceth a new Antecedent; and the Consequent of the Former by the Antecedent of the Latter; wihch gives a new Consequent. And therefore it is usually done by an Oblique Decussation of the Lines. For Example, If you would take 6 to 5 out of 4 to 3, you may set them down thus. Then 4 multiply'd by 5 makes 20; and 3 by 6 gives 18. So 20 to 18; *i. e.* 10 to 9, is the Remainder.

20. 28.
10. 9.

der. That is, Subſtract a Third *Minor* out of a Fourth, and there will remain a Tone *Minor*.

Multiplication of Rations is the ſame with their Addition; only it is not wont to be of divers Rations, but of the ſame, being taken twice, thrice, or oftener, as you pleaſe. And as before, in Addition, you added divers Rations by Multiplying them: So here, in Multiplication, you add the ſame Ration to it ſelf, after the ſame manner, *viz.* by Multiplying the Terms of the ſame Ratio by themſelves; *i. e.* the Antecedent by it ſelf, and the Conſequent by it ſelf (which in other words is to Multiply the ſame by 2) and will, in the Operation, be to Square the Ration firſt propounded (or give the Second Ordinal Power; the Ration firſt given being the Firſt Power or Side) And to this Product, if the Simple Ration ſhall again be added (after the ſame manner as before)

the

the Aggregate will be the Triple of the Ration firſt given; or the Product of that Ration Multiply'd by 3; *viz.* the Cube, or Third Ordinal Power. Its *Biquadrate*, or Fourth Power, proceeds from Multiplying it by 4; and ſo ſucceſſively in order as far as you pleaſe you may advance the Powers. For inſtance, The Duple Ration 2 to 1, being added to it ſelf, Dupled, or Multiply'd by 2, produceth 4 to 1 (the Ration *Quadruple*) and if to this, the firſt again be added (which is equivalent to Multiplying that ſaid firſt by 3) there will ariſe the Ration *Octuple*, of 8 to 1. Whence the Ration 2 to 1, being taken for a Root, its Duple 4 to 1, will be the Square; its Triple 8 to 1, the Cube thereof, *&c.* as hath been ſaid above. And to uſe another inſtance; To Duple the Ration of 3 to 2; it muſt be thus Squar'd; 3 by 3 gives 9; 2 by 2 gives 4.

So

So the Duple or Square of 3 to 2, is 9 to 4. Again, 9 by 3 is 27; and 4 by 2 is 8: So the Cubic Ration of 3 to 2 is 27 to 8. Again, to find the Fourth Power, or *Biquadrate*; (*i. e.* Squar'd Square) 27 by 3 is 81; 8 by 2 is 16: So 81 to 16 is the Ration of 3 to 2 Quadrupled; as 'tis Dupled by the Square, Tripled by by the Cube, *&c.* To apply this Instance to our present purpose; 3 to 2 is the Ration of *Diapente*, or a Fifth in Harmony; 9 to 4 is the Ration of twice *Diapente*, or a Ninth (*viz.* *Diapason* with Tone *Major*) 27 to 8 is the Ration of thrice *Diapente*, or three Fifths; which is *Diapason* with Six *Major* (*viz.* 13th *Major*) The Ration of 81 to 16 makes four Fifths, *i. e.* *Dis-diapason*, with two Tones *Major*; *i. e.* a Seventeenth *Major*, and a Comma of 81 to 80.

To

To Divide any Ration, you muſt take the contrary way; And by Extracting of theſe Roots Reſpectively, Diviſion by their *Indices* will be performed. *E. gr.* To Divide it by 2, is to take the Square Root of it; by 3, the Cubic Root; by 4, the Biquadratick, *&c.* Thus to divide 9 to 4, by 2; The Square Root of 9, is 3; the Square Root of 4, is 2: Then 3 to 2 is a Ration juſt half ſo much as 9 to 4.

From hence it will be obvious to any to make this Inference; That Addition and Multiplication of Rations are (in this Caſe) one and the ſame thing. And theſe Hints will be ſufficient to ſuch as bend their Thoughts to theſe kinds of Speculations, and no great Treſpaſs upon thoſe that do not.

The Advantage of proceeding by the Ordinal Powers, Square, Cube, *&c.* (as is before mentioned) may be very uſefull where there is occaſion of large

large Progreſſions. As to find (for Example) how many Comma's are contained in a Tone *Major*, or other Interval. Let it be, How many are in Diapaſon; Which muſt be done by Multiplying Comma's; *i. e.* Adding them, till you arrive at a Ration Equal to Octave (if that be ſought) *viz.* Duple. Or elſe by Dividing the Ration of Diapaſon, by that of a Comma, and finding the Quotient; which may be done by Logarithms. And herein I meet with ſome Differences of Calculations.

Merſennus finds, by his Calculation, $58\frac{1}{2}$ Comma's, and ſomewhat more in an Octave. But the late *Nicholas Mercator*, a Modeſt Perſon, and a Learned and Judicious Mathematician, in a Manuſcript of his, of which I have had a Sight; makes this Remark upon it. *In ſolvendo hoc Problemate aberrat Merſennus.* And He, working by the Logarithms, finds out

but

but 55, and a little more. And from thence has deduced an Ingenious Invention of finding and applying a leaſt Common Meaſure to all Harmonic Intervals; not preciſely perfect, but very near it.

Suppoſing a Comma to be $\frac{1}{53}$ part of Diapaſon; for better Accommodation rather than according to the true Partition $\frac{1}{55}$; which $\frac{1}{53}$ he calls an Artificial Comma, not exact, but differing from the true Natural Comma about $\frac{1}{20}$ part of a Comma, and $\frac{1}{1000}$ of Diapaſon (which is a Difference imperceptible) Then the Intervals within Diapaſon will be meaſured by Comma's according to the following Table. Which you may prove by adding two, or three, or more of theſe Numbers of Comma's, to ſee how they agree to conſtitute thoſe Intervals, which they ought to make; and the like by ſubſtracting.

Intervals

Intervals	$\frac{o}{53}$	Intervals	$\frac{o}{53}$
Comma	1	4th	22
Dieſis	2	*Tritone*	26
Semit. Minus	3	*Semidiapente*	27
Semit. Medium	4	5th	31
Semit. Majus	5	6th *Minor*	36
Semit. Max.	6	6th *Major*	39
Tone Minor	8	7th *Minor*	45
Tone Major	9	7th *Major*	48
3d *Minor*	14	*Octave*	53
3d *Major*	17		

This I thought fit, on this occaſion, to impart to the Reader, having leave ſo to doe from Mr. *Mercator's* Friend, to whom He preſented the ſaid Manuſcript.

Here I may advertiſe the Reader; that it is indifferent whether you compare the greater Term of an Harmonic Ration to the leſs, or the leſs to the greater;

greater; *i.e.* whether of them you place as Antecedent. *E.gr.* 3 to 2, or 2 to 3. Becauſe in Harmonics, the proportions of Lengths of Chords, and of their Vibrations are reciprocal or Counter-changed. As the Length is increaſed, ſo the Vibrations are in the ſame proportion decreaſed; *& è contra.* If therefore (as in *Diapente*) the length of the Uniſon String be 3, then the length (*cæteris paribus*) of the String which in aſcent makes *Diapente* to that Uniſon muſt be 2, or $\frac{2ds}{3}$. Thus the Ration of *Diapente* is 2 to 3 in reſpect of the length of it, compared to the length of the Uniſon String.

Again, the String 2 vibrates thrice, in the ſame time that the String 3 vibrates twice. And thus the Ration of *Diapente* in reſpect of Vibrations is 3 to 2. So that where you find in Authors, ſometimes the greater Number in the Rations ſet before and made the Antecedent, ſometimes ſet after

and

and made the Conſequent; You muſt underſtand in the former, the Ration of their Vibrations; and in the latter, the Ration of their Lengths; which comes all to one.

Or you may underſtand the Uniſon to be compared to *Diapente* above it, and the Ration of Lengths is 3 to 2; of Vibrations 2 to 3: or elſe *Diapente* to be compared to the Uniſon, and then the Ration of Lengths is 2 to 3; of Vibrations 3 to 2. This is true in ſingle Rations, or if one Ration be compared to another; then the two Greater Terms muſt be ranked as Antecedents: or otherwiſe, the two Leſs Terms.

The Difference between Arithmetical and Geometrical Proportion is to be well heeded. An Arithmetical mean Proportion is that which has Equal difference to the Antecedent and Conſequent Terms of thoſe Numbers to which it is the Mediety; and

is

is found by adding the Terms and taking half the Sum. Thus between 9 and 1, which added together make 10, the Mediety is 5; being Equidifferent from 9 and from 1; which Difference is 4: As 5 exceeds 1 by 4; ſo likewiſe 9 exceeds 5 by 4. And thus in Arithmetical Progreſſion 2, 4, 6, 8; where the Difference is onely conſidered, there is the ſame Arithmetical Proportion between 2 and 4, 4 and 6, 6 and 8; and between 2 and 6, and 4 and 8. But in Geometrical Proportion where is conſidered, not the Numerical Difference, but another Habitude of the Terms, *viz.* How many times, or how much of a time, or times, one of them doth contain the other (as hath been explained at large in the beginning of this Chapter.) There the Mean Proportional is not the ſame with Arithmetical, but found another way; and Equidifferent Progreſſions make different Rations. The Rations

(taking them all in their leaſt Terms) expreſſed by leſs Numbers, being greater than thoſe of greater Numbers, I mean in Proportions *Super-particular*, &c. Where the Antecedents are Greater than the Conſequents: (as on the Contrary, where the Antecedents are Leſs than the Conſequents, the *Ratio*'s of Leſs Numbers are Leſs than the *Ratio*'s of Greater.) The Mediety of 9 to 1, is not now 5, but 3; 3 having the ſame Ration to 1, as 9 has to 3 (as 9 to 3, ſo 3 to 1) *viz.* Triple. And ſo in Progreſſion Arithmetical, of Terms having the ſame Differences; if conſidered Geometrically, the Terms will all be comprehended by unequal Rations. The Differences of 2 to 4, 4 to 6, 6 to 8, are Equal; but the Rations are unequal: 2 to 4 is leſs than 4 to 6, and 4 to 6 leſs than 6 to 8. As on the Contrary; 4 to 2, is greater than 6 to 4; and 6 to 4 greater than 8 to 6. For 4 to 2

is Duple; 6 to 4 but *Sesquialtera* (one and a half onely, or $\frac{3}{2}$) and 8 to 6 is no more than *Sesquitertia*, (one and a Third part, or $\frac{4}{3}$) which shews a considerable Inequality of their Rations. In like manner, 6 to 2 is Triple; 8 to 4 is but Duple; and yet their Differences Equal. Thus the mean Rations comprehended in any greater Ration divided Arithmetically; *i. e.* by Equal Differences; are unequal to one another considered Geometrically. Thus 2, 3, 4, 5, 6; if you consider the Numbers, make an Arithmetical Progression: But if you consider the Rations of those Numbers, as is done in Harmony, then they are Unequal; every one being greater or less (according as you proceed by Ascent or Descent) than the next to it. Thus in this progression, 2 to 3 is the greatest, being *Diapente*; 3 to 4 the next, *Diatessaron*; 4 to 5 still less, *viz. Ditone*; 5 to 6 the least, being *Sesquitone*. Or, if you descend,

cend, 6 to 5 leaſt; 5 to 4 next, &c. Theſe are the mean Rations comprehended in the Ration of 6 to 2, by which *Diapaſon cum Diapente*, or a 12th, is divided into the aforeſaid Intervals, and meaſured by them: *viz.* as is 6 to 2, (*viz.* Triple.) So is the Aggregate of all the mean Rations within that Number; 6 to 5, 5 to 4, 4 to 3, and 3 to 2. Or 6 to 5, 5 to 2; or 6 to 4, 4 to 2; or 6 to 3, 3 to 2. The Aggregates of theſe are Equal to 6 to 2, *viz.* Triple.

This is premiſed in order to proceed to what was intimated in the foregoing Chapter.

Taking notice firſt of this procedure, peculiar to Harmonics; *viz.* To make Progreſſion or Diviſion in Arithmetical Proportion in reſpect of the Numbers; but to conſider the things Numbred according to their Rations Geometrical. And thus Harmonic proportion, is ſaid to be compounded of Arithmetical and Geometrical.

You

You may find them all in the Diviſion of the Syſteme of *Diapaſon*, into *Diapente* and *Diateſſaron*, i. e. 5th and 4th; aſcending from the Uniſon.

If by *Diapente* firſt, Then by 2, 3, 4, Arithmetically: If firſt by *Diateſſaron*, Then by 3, 4, 6, Harmonically. And theſe Rations conſidered Geometrically, in Relation to Sound; There is likewiſe found Geometrical Proportions between the Numbers 6, 3, to 4, 2; and 6, 4, to 3, 2.

The Antients therefore owning onely 8th, 5th, and 4th, for Simple Conſonant Intervals; concluded them all within the Numbers of 12, 9, 8, 6, which contained them all: *viz.* 12 to 6, *Diapaſon*; 12 to 8, *Diapente*; 12 to 9, *Diateſſaron*; 9 to 8, *Tone*. And which ſerved to expreſs the three Kinds of Proportion; *viz.* Harmonical, between 12 to 8, and 8 to 6; Arithmetical, between 12 to 9, and 9 to 6; and Geometrical, between 12 to 9, and 8 to 6;

and between 12 to 8, and 9 to 6. It was ſaid therefore, That *Mercurius* his Lyre was ſtrung with four Chords, having thoſe Proportions, 6, 8, 9, 12. *Gaſſend.*

I intimated that I would here more largely explain that ready and eaſie way of finding and meaſuring the mean Rations contained in any of thoſe Harmonick Rations given, by transferring them out of their Prime or Radical Numbers, into greater Numbers of the ſame Ration. By Dupling (not the Ration, but the Terms of it; ſtill continuing the ſame Ration) you will have one Mediety: as 2 to 1 Dupled is 4 to 2; and you have 3 the Mediety. By Tripling you will have two Means; 2 to 1 Tripled is 6 to 3, containing 3 Rations; 6 to 5, 5 to 4, 4 to 3; and ſo ſtill more, the more you multiply it.

Now obſerve, Firſt, That any Ration *Multiplex* or *Superpartient* (or by trans-

transferring it out of its Radical Numbers made like *Superpartient*) contains ſo many *Superparticular* Rations, as there are Units in the Difference between the Antecedent and the Conſequent. Thus in 8 to 4 (being 2 to 1 transferred by Quadrupling) the Difference is 4, and it contains 4 *Superparticular* Rations; *viz.* 8 to 7, 7 to 6, 6 to 5, and 5 to 4. Where though the Progreſſion of Numbers is Arithmetical, yet the Proportions of exceſs are Geometrical and Unequal. The *Superparticular* Rations expreſſed by leſs Numbers, being Greater, as hath been ſaid, than thoſe that conſiſt of Greater Numbers; 5 to 4 is a Greater Ration than 6 to 5, and 6 to 5 Greater than 7 to 6, and 7 to 6 than 8 to 7; as a Fourth part is Greater than a Fifth, and a Fifth Greater than a Sixth, *&c.* But in this Inſtance, there are two Rations not appertaining to Harmonics; *viz.* 8 to 7, and 7 to 6.

Secondly therefore, you may make unequal Steps, and take none but Harmonick Rations, by Selecting Greater and fewer intermediate Rations, tho' ſome of them compoſed of ſeveral *Superparticulars*; provided you do not diſcontinue the Rational Progreſſion, but that you repeat ſtill the laſt Conſequent, making it the next Antecedent. As if you meaſure the Ration of 8 to 4, by 8 to 6, and 6 to 4; or by 8 to 5, and 5 to 4; or by 8 to 6, and 6 to 5, and 5 to 4. In theſe three ways the Rations are all Harmonical, and are reſpectively contained in, and make up the Ration of 8 to 4. Thus you may meaſure, and divide, and compound moſt Harmonick Rations without you Pen.

To that End, I would have my Reader to be very perfect in the Radical Numbers, which expreſs the Rations of the Seven firſt (or uncompounded) Conſonants: *viz.* *Diapaſon*, 2 to 1;

Dia-

Diapente, 3 to 2; *Diateſſaron*, 4 to 3; *Ditone*, 5 to 4; *Trihemitone*, 6 to 5; *Hexachordon Majus*, 5 to 3; *Hexachordon Minus*, 8 to 5. And likewiſe of the Degrees in Diatonick Harmony, viz. *Tone Major*, 9 to 8; *Tone Minor*, 10 to 9; *Hemitone Major*, 16 to 15. And the Differences of thoſe Degrees; *Hemitone Greateſt*, 27 to 25; and *Hemitone Minor*, 25 to 24; *Comma*, or *Schiſm*, 81 to 80; *Dieſis Enharmonic*, 128 to 125.

Of other *Hemitones*, I ſhall treat in the Eighth Chapter.

Now if you would divide any of the Conſonants into two Parts, you may do it by the Mean, or Mediety of the two Radical Numbers; if they have a Mean: And where they have not (as when their *Ratio*'s are *Superparticular*) you need but Duple thoſe Numbers, and you will have a Mean (one or more.) Thus Duple the Numbers of the Ration of *Diapaſon*, 2 to 1;

and

and you have 4 to 2; and then 3 is the Mean by which it is divided into two Unequal, but Proper and Harmonical parts; *viz.* 4 to 3, and 3 to 2. After this manner *Diapaſon*, 4 to 2; comprehends 4 to 3, and 3 to 2. So *Diapente*, 6 to 4; is 6 to 5, and 5 to 4. *Ditone*, 10 to 8; is 10 to 9, and 9 to 8. So Sixth *Major*, 5 to 3; is 5 to 4, and 4 to 3.

Though, from what was now obſerv'd, you may divide any of the Conſonants into intermediate Parts; yet when you divide theſe three following, *viz.* Sixth *Minor*, *Diateſſaron*, and *Tribemitone*; you will find that thoſe Parts into which they are divided, are not all ſuch Intervals as are Harmonical. The Sixth *Minor*, whoſe Ration is 8 to 5, contains in it three Means; *viz.* 8 to 7, 7 to 6, and 6 to 5; the laſt whereof onely is one of the Harmonick Intervals, of which the Sixth *Minor* conſiſts; *viz.* *Tribemitone*: and

to

to make up the other Interval, *viz.* *Diateſſaron*; you muſt take the other two, 8 to 7, and 7 to 6; which being added (or, which is the ſame thing, taking the *Ratio* of their two Extream Terms, *That* being the Sum of all the Intermediate ones added) you have 8 to 6, or (in the leaſt Terms) 4 to 3. Again *Diateſſaron*, in Radical Numbers, 4 to 3; being (if thoſe Numbers are dupled) 8 to 6, gives for his Parts, 8 to 7, and 7 to 6; which *Ratio's* agree with no Intervals that are Harmonick. Therefore you muſt take the Ration of *Diateſſaron* in other Terms, which may afford ſuch Harmonick Parts. And to do this, you muſt proceed farther than dupling (or adding it once to it ſelf) for to this Duple you muſt add the firſt Radical Numbers once again (which in effect is the ſame with Tripling it at firſt) *viz.* 4 and 3, to 8 and 6; and the Aggregate will be a

new,

new, but Equivalent, Ration of *Diateſſaron*; viz. 12 to 9. And this gives you three Means, 12 to 11, and 11 to 10; both Unharmonical; but, which together are, as was ſhewed before, the ſame with 12 to 10 (or 6 to 5) *Trihemitone*; and 10 to 9, *Tone Minor*: and are the two Harmonical Intervals of which *Diateſſaron* conſiſts, and which divide it into the two neareſt Equal Harmonick Parts. Laſtly *Trihemitone*, or Third *Minor*, 6 to 5; or (thoſe Numbers being dupled) 12 to 10, gives 12 to 11, and 11 to 10; which are Unharmonical Rations: but Tripled (after the former manner) 6 to 5 gives 18 to 15; which divides it ſelf (as before) into 18 to 16, *Tone Major*; and 16 to 15, *Hemitone Major*.

Thus by a little Practice all Harmonick Intervals will be moſt eaſily meaſured, by the leſſer Intervals comprized in them. Now, (for exerciſe ſake)

ſake) take the Meaſures of a greater Ration: Suppoſe that of 16 to 3 be given as an Harmonick Syſteme. To find what it is, and of what Parts it conſiſts: Firſt find the groſs Parts, and then the more Minute. You will preſently judge, that 16 to 8 (being a Part of this Ration) is *Diapaſon*; and 8 to 4 is likewiſe *Diapaſon*: then 16 to 4 is *Diſdiapaſon*, or a Fifteenth; and the remaining 4 to 3, is a Fourth. So then, 16 to 3, is *Diſdiapaſon*, and *Diateſſaron*; i. e. an Eighteenth: 16 to 8, 8 to 4, and 4 to 3.

But

But to find all the Harmonick Intervals within that Ration (for we now consider Rations as relating to Harmony) take this Scheme.

16 to 3 contains,

In Radicals.

16 to 15,		*Hemitone.*
15 to 12,	5 to 4,	*Ditone.*
12 to 10,	6 to 5,	*Trihemitone.*
10 to 9,		*Tone Minor.*
9 to 8,		*Tone Major.*
8 to 6,	4 to 3,	*Diateſſaron.*
6 to 5,		*Trihemitone.*
5 to 4,		*Ditone.*
4 to 3,		*Diateſſaron.*
Tot. 16 *to* 3.		*Diſdiapaſon cum Diateſſaron.*

Or Thus,

In Radicals.

16 to 10,	8 to 5,	6th *Minor.*
10 to 6,	5 to 3,	6th *Major.*
6 to 4,	3 to 2,	5th
4 to 3,		4th
Tot. 16 *to* 3.		*Eighteenth.*

All

All these Invervals thus put together are comprehended in, and make up the Ration of 16 to 3; being taken in a Conjunct Series of Rations.

But otherwise, within this Compass of Numbers are contained many more Expressions of Harmonick Ration. *Ex. gr.*

	Radicals.		*Radicals.*
16 to 15.		12 to 6,	2 to 1.
16 to 12,	4 to 3.	12 to 4,	3 to 1.
16 to 10,	8 to 5.	12 to 3,	4 to 1.
16 to 8,	2 to 1.	10 to 9.	
16 to 6,	8 to 3.	10 to 8,	5 to 4.
16 to 4,	4 to 1.	10 to 6,	5 to 3.
16 to 3.		10 to 5,	2 to 1.
15 to 12,	5 to 4.	9 to 8.	
15 to 10,	3 to 2.	9 to 6,	3 to 2.
15 to 5,	3 to 1.	9 to 3,	3 to 1.
15 to 3,	5 to 1.	8 to 6,	4 to 3.
14 to 7,	2 to 1.	8 to 5,	
12 to 10,	6 to 5.	8 to 4,	2 to 1.
12 to 9,	4 to 3.	6 to 5, *&c.*	
12 to 8,	3 to 2.	*Vid.* Pag. 67.	

And now I suppose the Reader better prepared to proceed in the remainder of this Discourse, where we shall treat of Discords.

CHAP.

CHAP. VI.

Of Diſcords and Degrees.

ALL Habitudes of one Chord to another, that are not Concords (ſuch as are before deſcribed) are Diſcords; which are, or may be innumerable, as are the minute Tenſions by which one Chord may be made to vary from it ſelf, or from another. But here we are to conſider onely ſuch Diſcords as are uſeful (and in truth neceſſary) to Harmony, or at leaſt relating to it, as are the Differences found between Harmonick Intervals.

And theſe apt and uſeful Diſcords, are either Simple uncompounded Intervals, ſuch as immediately follow one another, aſcending or deſcending in

in the Scale of Muſic: As *Ut Re Mi. Fa Sol La Fa Sol*, and are called Degrees: Or elſe, greater Spaces or Intervals compounded of Degrees including or skipping over ſome of them, as all the Concords do, *Ut Mi*, *Ut Fa*, *Ut Sol*, &c. And ſuch are the Diſcords of which we now treat, as principally the *Tritone*, Falſe Fifth, and the two Sevenths; *Major*, and *Minor*, if they be not rather among the Degrees, &c. For more perſpicuity I ſhall treat of them ſeverally; *viz.* of Degrees, of Diſcords, and of Differences.

And Firſt of Degrees.

Degrees, are uncompounded Intervals, which are found upon 8 Chords and in 7 Spaces, by which an immediate Aſcent or Deſcent is made from the *Uniſon* to the *Octave* or *Diapaſon*; and by the ſame progreſſion to as many *Octaves* as there may be occaſion.

These are different, according to the different Kinds of Music; *viz.* *Enharmonic*, *Chromatic*, and *Diatonic*, and the several Colours of the two Latter. (All which I shall more conveniently explain by and by.) But of these now mentioned, the *Diatonic* is the most Proper and Natural Way: The other two, if for Curiosities sake we consider them only by running the Notes of an *Octave* up or down in these Scales, seem rather a force upon Nature; yet herein probably might lye the Excellency of the Ancient *Greeks*. But we now use only the *Diatonic* kind, intermixing here and there some of the *Chromatic*, (and more rarely some of the *Enharmonic*:) And our Excellency seems to lye in most artificial Composing, and joyning several parts in Symphony or Consort; which they cannot be supposed to have effected, at least in so many Parts as we ordinarily make; because (as is generally

rally affirmed of them) they owned no Concords, beſides Eighth, Fifth, and Fourth, and the Compounds of theſe.

F. Kircher (cited alſo by *Gaſſendus* without any Mark of Diſſent) is of Opinion, That the Ancient *Greeks* never uſed Conſort Muſic, *i. e.* of different Parts at once; but only Solitary, for one ſingle Voice, on Inſtrument. And that *Guido Aretinus* firſt invented and brought in Muſic of Symphony or Conſort both for the one and the other. They applyed Inſtruments to Voice, but how they were managed, He muſt be wiſer than I, that can tell.

This way of theirs ſeems to be more proper (by the Elaborate Curioſity and Nicety of Contrivance of Degrees, and by Meaſures, rather than by Harmonious Conſonancy, and by long ſtudied performance) to make great Impreſſions upon the Fancy, and operate accordingly, as ſome Hiſtories

relate: Ours, more Sedately affects the Underſtanding and Judgment from the judicious Contrivance, and happy Compoſition of Melodious Conſort. The One quietly, but powerfully affects the Intellect by true Harmony: The Other, chiefly by the *Rythmus*, violently attacks and hurries the Imagination. In fine, upon the Natural Grounds of Harmony (of which I have hitherto been treating) is founded the *Diatonic* Muſic; but not ſo, or not ſo regularly, the *Chromatic* and *Enharmonic* kinds. Take this following view of them.

The Ancients aſcended from the *Uniſon* to an *Octave* by two Syſtemes of Tetrachords or Fourths. Theſe were either Conjunct, when they began the Second Tetrachord at the Fourth Chord; *viz.* with the laſt Note of the Firſt Tetrachord; and which being ſo joyned, conſtituted but a Seventh: And therefore they aſſumed a *Tone* beneath

neath the *Uniſon* (which they therefore called *Proſlambanomenos*) to make a full Eighth.

Or elſe the two Tetrachords were diſjunct; the Second taking its beginning at the Fifth Chord; there being always a *Tone Major* between the Fourth and Fifth Chords. So, the Degrees were immediately applyed to the 4ths, and by them to the *Octave*; and were different according to the different Kinds of Muſic. In the common *Diatonic Genus*, the Degrees were *Tone* and *Semitone*; Intervals more Equal and Eaſy, and Natural. In the common *Chromatic*, where the Degrees were *Hemitones* and *Trihemitones*; the Difference of ſome of the Intervals was Greater. But the Greateſt Difference, and conſequently Difficulty, was in the *Enharmonic* Kind, uſing only *Dieſis*, or Quarter of a *Tone*, and *Ditone*; as the Degrees whereby they made the Tetrachord.

And to conſtitute theſe Degrees, ſome of them, *viz.* the Followers of *Ariſtoxenus*, divided a *Tone Major* into 12 Equal Parts; *i. e.* Suppoſed it ſo divided: Six of which being the *Hemitone*, (*viz.* half of it,) made a Degree of *Chromatic Toniæum*. And Three of them, or a Quarter called *Dieſis*; a Degree *Enharmonic*. The *Chromatic* Fourth roſe thus, *viz.* from the Firſt Chord to the Second was a *Hemitone*; from the Second to the Third, a *Hemitone*; from the Third to the Fourth, a *Trihemitone*; or as much as would make up a juſt Fourth. And this laſt Space (in this caſe) was accounted as well as either of the other, but one Degree or undivided Interval. And they called them *Spiſs* Intervals (πυκνὰ) when two of thoſe other Degrees put together, made not ſo great an Interval as one of theſe; as, in the *Enharmonic* Tetrachord, two *Dieſes* were leſs than the remaining *Ditone*, and in the com-

common *Chromatic*, two *Hemitone* Degrees were leſs than the remaining *Trihemitone* Degree.

Then for the *Enharmonic* Fourth, the firſt Degree was a *Dieſis*, or Quarter of a *Tone*; the Second alſo 3 of thoſe 12 parts, *viz.* a *Dieſis*; the Third a *Ditone*; ſuch as made up a juſt Fourth. And this *Ditone*, (though ſo large a Degree) being conſidered as thus placed (in the *Enharmonic* Tetrachord) was likewiſe to them but as one uncompounded or entire Interval.

Theſe were the Degrees *Chromatic*, and *Enharmonic*. Though they alſo might be placed otherwiſe, *i. e.* The greater Degree in theſe may change its place, as the *Hemitone*, (the leſs Degree) doth in the *Diatonic Genus*. And from this change chiefly aroſe the ſeveral Moods, *Dorian*, *Lydian*, &c. From all which, their Muſic no doubt (though it be hard to us to conceive) muſt afford extraordinary delight and pleaſure;

if it did bear but a reaſonable Proportion to their infinite Curioſity and Labour. And as we may ſuppoſe ie to have differed very much from that which now is, and for ſeveral Ages hath been uſed: So conſequently we may look upon it as in a manner loſt to us.

In proſecution of my Deſign I am only, or chiefly to inſiſt on the other Kind of Degrees; which are moſt proper to the Natural Explanation of Harmony; *viz.* the Degrees *Diatonic*; which are ſo called; not becauſe they are all *Tones*; but becauſe moſt of them, as many as can be, are ſuch; *viz.* in every *Diapaſon*, 5 *Tones*, and two *Hemitones*. Upon theſe I ſay I am to inſiſt, as being, of thoſe before mentioned, the moſt Natural and Rational.

Di-

Digreſſion.

But before we proceed, it may perhaps be a ſatisfaction to the Reader, after what has been ſaid, to have a little better Proſpect of the Ancient Greek Muſic, by ſome general Account; not of their whole Doctrine, but of that which relates to our preſent Subject, *viz.* their Degrees, and Scales of Harmony, and Notes.

Firſt then, take out of *Euclid* the Degrees according to the three *Genera*; which were, *Enharmonic*, *Chromatic*, and *Diatonic*; which Kinds have ſix Colours (as they call'd them.) *Euclid, Introd. Harm. Pag.* 10.

The *Enharmonic* Kind had but one Colour; which made up its Tetrachord by theſe Intervals; a *Dieſis* (or Quarter of a *Tone*,) then ſuch another *Dieſis*; and alſo a *Ditone* incompoſit.

The *Chromatic* had three Colours; by which it was divided into *Molle*, *Seſcuplum*, and *Toniæum*. 1ſt. *Molle*,

1st. *Molle*, in which the Tetrachord roſe by a Triental *Dieſis* (four of thoſe 12 parts mentioned before) or third part of a *Tone*; and another ſuch *Dieſis*; and an Incompoſit Interval, containing a *Tone*, and half, and third part of a *Tone*: and it was called *Molle*, becauſe it hath the leaſt, and conſequently moſt Enervated *Spiſs* Intervals within the *Chromatic Genus*.

2d. *Seſcuplum*; by a *Dieſis* which is *Seſquialtera* to the *Enharmonic Dieſis*, and another ſuch *Dieſis*, and an Incompoſit Interval of 7 *Dieſes* Quadrantal; *viz.* Each being 3 Duodecimals of a *Tone*.

3d. *Toniæum*; by a *Hemitone*, and *Hemitone*, and *Trihemitone*; and is called *Toniæum*, becauſe the two *Spiſs* Intervals make a *Tone*. And this is the ordinary *Chromatic*.

The *Diatonick* had 2 Colours; it was *Molle*, and *Syntonum*.

1st. *Molle*;

1st. *Molle*; by a *Hemitone*, and an incompoſit Interval of 3 Quadrantal *Dieſes*, and an Interval of 5 ſuch *Dieſes*.

2d. *Syntonum*, by a *Hemitone*, and a *Tone*, and a *Tone*. And this is the common *Diatonic*.

To underſtand this better, I muſt re-aſſume ſomewhat which I mentioned, but not fully enough before. A *Tone* is ſuppoſed to be divided into 12 leaſt parts, and therefore a *Hemitone* contains 6 of thoſe Duodecimal (or Twelfth) parts of a *Tone*; a *Dieſis Trientalis* 4; *Dieſis Quadrantalis* 3; The whole *Diateſſaron* 30. And the *Diateſſaron* in each of the 3 Kinds, was made and perform'd upon 4 Chords, having 3 mean Intervals of Degrees, according to the following Numbers and Proportions of thoſe 30 Duodecimal parts.

Enharmonic, by 3, and 3, and 24.

Chromatic, *Molle*, by 4, and 4, and 22.
Hemiolion, or *Sesquiplum*, by $4\frac{1}{2}$, and $4\frac{1}{2}$, and 21.
Toniæum. by 6, and 6, and 18.

Diatonic, *Molle*, by 6, and 9, and 15.
Syntonum, by 6, and 12, and 12.

To each of these Kinds, and the Moods of them, they fitted a perfect Systeme, or Scale of Degrees to *Disdiapason*; as in the following Example taken out of *Nichomachus*: To which I have prefixed our Modern Letters.

E. Nichomacho; Pag. 22.

A	*Nete Hyperbolæon.*	
G	*Paranete Hyperbolæon.*	*Enh. Chro. Diat.*
F	*Trite Hyperbolæon.*	*Enh. Chro. Diat.*
E	*Nete Diezeugmenon.*	
D	*Paranete Diezeugmenon.*	*Enh. Chro. Diat.*
C	*Tri-*	

C	*Trite Diezeugmenon.*	*Enh. Chro. Diat.*
B	*Parameſe.*	
D	*Nete Synemmenon.*	
C	*Paranete Synemmenon.*	*Enh. Chro. Diat.*
B	*Trite Synemmenon.*	*Enh. Chro. Diat.*
A	*Meſe.*	
G	*Lichanos Meſon.*	*Enh. Chro. Diat.*
F	*Parypate Meſon.*	*Enh. Chro. Diat.*
E	*Hypate Meſon.*	
D	*Lichanos Hypaton.*	*Enh. Chro. Diat.*
C	*Parypate Hypaton.*	*Enh. Chro. Diat.*
B	*Hypate Hypaton.*	
A	*Proſlambanomenos.*	

In this Scale of *Diſdiapaſon*, you ſee the *Meſe* is an *Octave* below the *Nete Hyperbolæon*, and an *Octave* above the *Proſlambanomenos*: And the *Lichanos*, *Parypate*, *Paranete*, and *Trite*, are changeable; as upon our Inſtruments are the Seconds, and Thirds, and Sixths, and

and Sevenths: The *Proſlambanomenos*, *Hypate*, *Meſe*, *Parameſe*, and *Nete*, are Immutable; as are the Uniſon, Fourths, Fifths, and Octaves.

Now from the ſeveral changes of theſe Mutable Chords, chiefly ariſe the ſeveral Moods (ſome call'd them *Tones*) of Muſic, of which *Euclid* ſets down Thirteen; to which were joyned two more, *viz.* *Hyperæolian* and *Hyperlydian*; and afterwards Six more were added.

I ſhall give you for a Taſt *Euclid's* Thirteen Moods. *Euclid.* Pag. 19.

Hypermixolydius, ſive *Hyperphrygius.*
Mixolydius acutior, ſive *Hyperiaſtius.*
Mixolydius gravior, ſive *Hyperdorius.*
Lydius acutior.
Lydius gravior, ſive *Æolius.*
Phrygius acutior.
Phrygius gravior, ſive *Iaſtius.*
Dorius.
Hypolydius acutior.

Hypo-

Hypolydius gravior, ſive *Hypoæolius.*
Hypophrygius acutior.
Hypophrygius gravior, ſive *Hypoiaſtius.*
Hypodorius.

Of theſe the moſt Grave, or Loweſt, was the *Hypodorian* Mood; the *Proſlambanomenos* whereof was fixed upon the loweſt clear and firm Note, of the Voice or Inſtrument that was to expreſs it; And then all along from *Grave* to *Acute* the Moods took their Aſcent by *Hemitones*, each Mood being a *Hemitone* higher or more acute than the next under it. So that the *Proſlambanomenos* of the *Hypermixolydian* Mood, was juſt an Eighth higher than that of the *Hypodorian*, and the reſt accordingly.

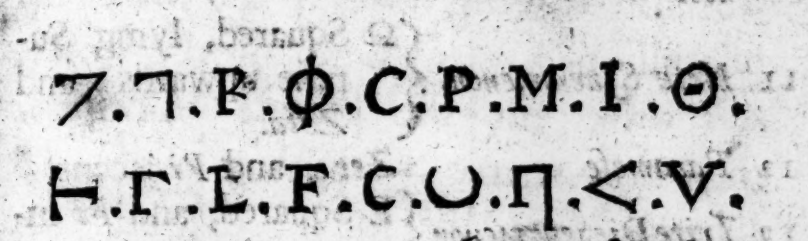
Now each particular Chord in the preceding Scale had two Signs or Notes [σημεία] by which it was characterized or deſcribed in every one of theſe Moods reſpectively, and alſo for

all

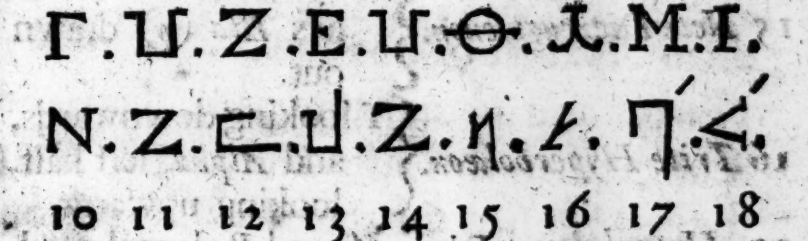
all the Moods in the ſeveral Kinds of Muſic; *Enharmonic*, *Chromatic*, and *Diatonic*; of which two Notes, the upper was for reading [λέξις] the lower for percuſſion [κρόυσις] One for the Voice, the other for the Hand. Conſider then how many Notes they uſed; 18 Chords ſeverally for 13 Moods (or rather 15, taking in the *Hyperæolian*, and *Hyperlydian*, which are all deſcribed by *Alypius*) and theſe ſuited to the three Kinds of Muſic. So many Notes, and ſo appropriated, had the Scholar then to learn and conn, who ſtudied Muſic. Of theſe I will give you in part a View out of *Alypius*.

Notes

Notes of the *Lydian* Mood in the *Diatonic Genus*.



1 2 3 4 5 6 7 8 9



10 11 12 13 14 15 16 17 18

1 *Proſlambanomenos.* { *Zeta* imperfect, and *Tau* jacent.
2 *Hypate Hypaton.* { *Gamma* averted, and *Gamma* right.
3 *Parypate Hypaton.* { *Beta* imperfect, and *Gamma* inverted.
4 *Hypaton Diatonos.* *Phi*, and *Digamma*.
5 *Hypate Meſon.* *Sigma*, and *Sigma*.
6 *Parypate Meſon.* *Rho*, and *Sigma* inverted.
7 *Meſon Diatonos.* *My*, and *Pi* drawn out.
8 *Meſe.* *Iota*, and *Lambda* jacent.

9 *Trite Synemmenon.* { *Theta*, and *Lambda* inverted.

10 *Synemmenon Diatonos.* *Gamma*, and *Ny*.

11 *Nete Synemmenon.* { Ω Squared, lying Supine upwards; and *Zeta*.

12 *Parameſe.* *Zeta*, and *Pi* jacent.

13 *Trite Diezeugmenon.* { E Squared, and *Pi* inverted.

14 *Diezeugmenon Diatonos.* Ω Squared, Supine, and *Zeta*.

15 *Nete Diezeugmenon.* { *Phi* jacent, and a careleſs *Eta* (ɴ) drawn out.

16 *Trite Hyperbolæon.* { ϒ looking downwards, and *Alpha*, left half, looking upwards.

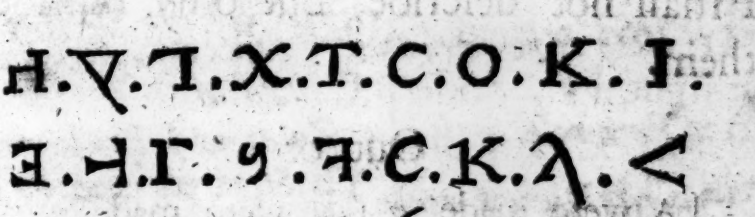
17 *Hyperbolæon Diatonos.* *My*, and *Pi* lengthened, with an Acute above.

18 *Nete Hyperbolæon.* { *Iota*, and *Lambda* jacent, with an Acute above.

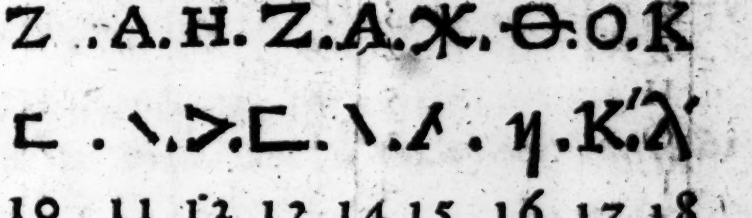
The Numeral Figures I have added under the Signs (or Marks) only for Reference to the Names of the Notes ſignified by them, to ſave deſcribing them twice.

Notes

Notes of the *Æolian* Mood in the *Diatonic Genus.*



1 2 3 4 5 6 7 8 9



10 11 12 13 14 15 16 17 18

1 *Proslambanomenos.* { *Eta* (H) imperfect averted, and E Quadrate averted.

2 *Hypate Hypaton*, &c. { *Delta* inverted, and *Tau* jacent, averted, &c.

Aristides (Pag. 91.) enumerates and describes all the Variations of every Letter in the Greek Alphabet; by which the Signs or Notes above mentioned,

and thoſe of the other Moods, were contrived out of them. They are in all 91; including the Proper Letters; I ſhall not deſcribe, but only number them.

Out of

Α were made	7	Ν were made	2
Β	2	Ξ	2
Γ	7	Ο	2
Δ	4	Π	7
Ε	3	Ρ	2
Ζ	2	Σ	6
Η	5	Τ	4
Θ	2	Υ	3
Ι	4	Φ	4
Κ	3	Χ	4
Λ	5	Ψ	2
Μ	5	Ω	4
	49		42
	91		

I ſhall only add a word or two concerning their Antient uſe of the Words *Diaſtem* and *Syſtem*. *Diaſtem* ſignifies an Interval or Space; *Syſtem* a Conjunction

junction or Compoſition of Intervals. So that generally ſpeaking, an *Octave*, or any other *Syſtem*, might be truly called a *Diaſtem*, and very frequently uſed to be ſo called, where there was no occaſion of Diſtinction. Though a *Tone*, or *Hemitone*, could not be called a *Syſtem*: For when they ſpoke ſtrictly, by a *Diaſtem* they underſtood only an Incompoſit Degree, whether *Dieſis*, *Hemitone*, *Tone*, *Seſquitone*, or *Ditone*; for the two laſt were ſometimes but Degrees, one *Enharmonic*, the other *Chromatic*. By *Syſtem* they meant, a Comprehenſive Interval, compounded of Degrees, or of leſs *Syſtems*, or of both. Thus a *Tone* was a *Diaſtem*, and *Diateſſaron* was a *Syſtem*, compounded of Degrees, or of a 3d. and a Degree. *Diapaſon* was a *Syſtem*, compounded of the leſſer *Syſtems*, 4th, and 5th; or 3d. and 6th; or of a Scale of Degrees: and the Scale of Notes which they uſed, was their Greateſt, or Perfect *Syſtem*. Thus with

them, a 3d. *Major*, and a 3d. *Minor*, in the *Diatonic Genus*, were (properly ſpeaking) *Syſtems*; the former being compounded of two *Tones*, and the latter of three *Hemitones*, or a *Tone* and *Hemitone*: But in the *Enharmonic* Kind, a *Ditone* was not a *Syſtem*, but an Incompoſit Degree; which, added to two *Dieſes*, made up the *Diateſſaron*: And in the *Chromatic* Kind, a *Trihemitone* was the like; being only an Incompoſit *Diaſtem*, and not a *Syſtem*.

But to return from this Digreſſion (which is not ſo much to my purpoſe, as to gratify the Reader's Curioſity) and continue our Diſcourſe according to Nature's Guidance, upon the *Diatonic* Degrees. It was ſaid that there are 5 *Tones* and 2 *Hemitones* in every *Diapaſon*. Now the reaſon why there muſt be 2 *Hemitones*, is, becauſe an Eighth is Naturally compoſed of, and divided into 5th. and 4th; and a Fifth is 3 *Tones* and a half; a Fourth 2 *Tones* and a half;

and

and the Aſcent, by Degrees, muſt paſs by Fourth and Fifth; which are always unchangeable, and keep the ſame Diſtance from *Uniſon*; and a juſt *Tone Major* of 9 to 8 always between them. Therefore the *Diapaſon* has not an Aſcent of 6 *Tones*; but of 5 *Tones* and 2 *Hemitones*, One *Hemitone* being placed in each Fourth Disjunct; in either of which Fourths, the Degrees may be altered by placing the *Hemitone* in the Firſt, or Second, or Third Degree of either. As, *MI, FA, Sol, La. La, MI, FA, Sol. Sol, La, MI, FA.* If this be done in the former *Tetrachord*, then is changed the Second, or Third *Chord*; If in the other Disjunct *Tetrachord*, then the Sixth, or Seventh is changed: The Fourth and Fifth being Stable and Immutable. By them we Naturally divide the *Diapaſon*: The Second, Third, Sixth, and Seventh, are alterable, as *Minor*, and *Major*, according to the place of the *Hemitone*.

Theſe *Tones* and *Hemitones* thus placed, are the Degree or Notes, by which an Aſcent or Deſcent is made from the *Uniſon* to the *Octave*, or through any other *Syſtem*; giving all the Concords their juſt Meaſures or Rations; and without which, we could neither Meaſure, nor Divide, nor well Practiſe to learn the greater Intervals, or Syſtems.

As we Naturally by the Judgment of our Ear, own, and reſt in the *Octave*, as the chief Conſonant; ſo we do as Naturally (without Study or Skill in Muſic) meaſure the *Syſtem* of a *Diapaſon* by theſe *Diatonic* Degrees; and can doe no otherwiſe. We cannot with our Voice, without infinite Study, frame to run up or down 8 Notes, without ſuch a Mixture of *Tones* and *Hemitones*; and we doe it eaſieſt, when we avoid *Tritones*. We ſee it in a Ring of Bells, of which the compleateſt and moſt pleaſant is a Peal of Six; which are beſt ſorted to have the *Hemitone* in the midſt;

i. e. between the Third and Fourth, both in Aſcending and Deſcending; and then there will be no *Tritone*: Ex. gr. *La, Sol, Fa, Mi, Re, Ut.* Where all Aſcents and Deſcents are made by juſt *Diateſſarons*. *Ut, Re, Mi, Fa. Re, Mi, Fa, Sol. Mi, Fa, Sol, La.* Or downwards; *La, Sol, Fa, Mi. Sol, Fa, Mi, Re. Fa, Mi, Re, Ut.*

And this is ſo Natural that it pleaſeth all Ears; and if they ſhould be diſpoſed in any other Order, it would be ſo diſagreeable, that any Ruſtick or unlearned Ear, of ſuch as know not what a *Tritone* is, would be able to judge, and find a diſlike of it. But then, how much more, if the Ring of Bells were diſpoſed by *Chromatic*, or *Enharmonic* Degrees, conſtituting the *Diateſſarons*? how abſurd and uncouth it would appear! The practiſe of thoſe kinds therefore, and in ſuch a manner, ſeems to be (as has been ſaid) a Violence upon Nature, and only for Curioſity.

In

In *Diatonic* Muſic, there is but one Sort of *Hemitone* amongſt the Degrees, called *Hemitone Major*; whoſe Ration is 16 to 15: being the Difference, and making a Degree between a *Tone Major*, and Third *Minor*; or between a Third *Major*, and a Fourth.

There are two Sorts of *Tones*; viz. *Major*, and *Minor*. *Tone Major* (9 to 8) being the Difference between a Fourth and Fifth: And *Tone Minor* (10 to 9) which is the Difference between Third *Minor* and Fourth. But both the *Tones* ariſing (as hath been ſaid) out of the Partion of a Third *Major*, in like manner as 5th. and 4th. do by the Partition of an Eighth: I may (with ſubmiſſion) make the following Remark; wherein, if I be too bold, or be miſtaken, I ſhall beg the Reader's pardon.

The Ancient Greek Maſters found out the *Tone* by the Difference of a Fourth and Fifth, ſubſtracting one from the other. But had they found it alſo

(and

(and that more Naturally) by the Diviſion of a Fifth; firſt into a *Ditone* and *Seſquitone*, and then by the like proper Diviſion of a true *Ditone* (or Third *Major*) into its proper parts; they muſt have found both *Tone Major*, and *Tone Minor*. *Euclid* reſts ſatisfied, That, *Inter ſuper-particulare non cadit Medium*. A ſuper-particular Ration cannot have a Mediety; *viz.* in whole Number: which is true in its Radical Numbers. But had he doubled the Radical Terms of a Super-particular, he might have found Mediums moſt Naturally and Uniformly dividing the Syſtems of Harmony. *Ex. gr.* The Duple Ration 2 to 1, as the Exceſs is but by an Unity; has the Nature of Super-particular: but 2 to 1, the Terms being dupled, is 4 to 2; where 3 is a Medium, which divides it into 4 to 3 (4th.) and 3 to 2 (5th.) Again, 3 to 2, dupling each Term, is 6 to 4; and in the ſame Manner gives the 2 Thirds, *viz.* 6 to 5, (3d. *Minor*)

and

and 5 to 4, (3d. *Major*.) Likewiſe the 3d. *Major*, 5 to 4, dupled as before, 10 to 8, give the 2 *Tones*; *i. e.* 10 to 9, *Tone Minor*, and 9 to 8, *Tone Major*.

And it ſeems to be a reaſon why the Antients did not diſcover and uſe the *Tone Minor*, and conſequently not own the *Ditone* for a Concord; becauſe They did not purſue this way of dividing the *Syſtems*. Although *Euclid* had a fair Hint to ſearch further, when he meaſured the *Diapaſon* by 6 *Tones* [*Major*] and found them to exceed the Interval of *Diapaſon*.

The *Pythagoreans*, not uſing *Tone Minor*, but two Equal *Tones Major*, in a Fourth, were forced to take a leſſer Interval for the *Hemitone*; which is called their *Limma*, or *Pythagorean Hemitone*; and, which added to thoſe two *Tones*, makes up the Fourth: it is a *Comma* leſs than *Hemitone Major*, (16 to 15;) and the Ration of it, is 256 to 243.

Yet

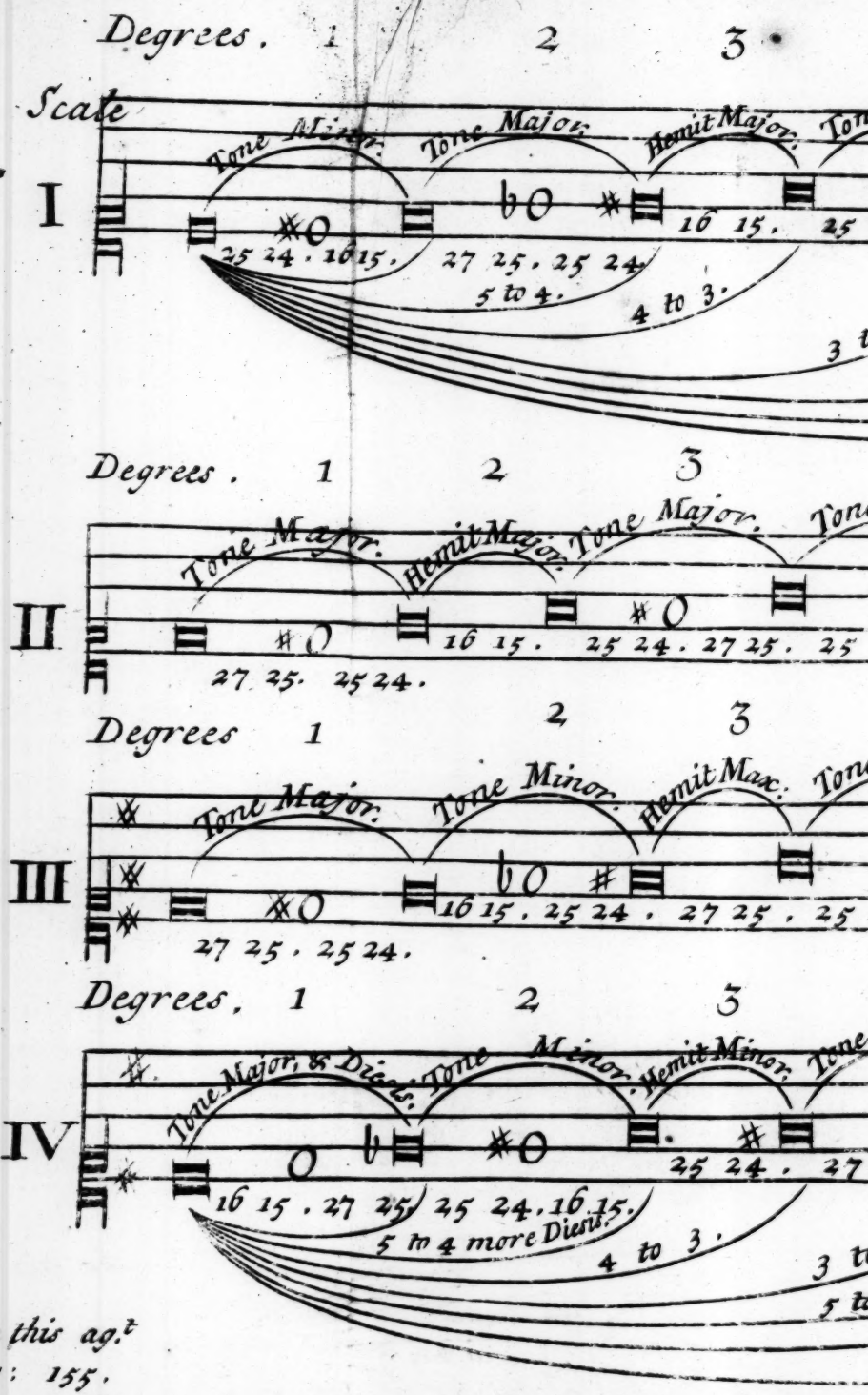
Yet we find the later Greek Maſters, *Ptolemy*, to take Notice of *Tone Minor*; and *Ariſtides Quintilianus*, to divide a *Seſquioctave Tone* (9 to 8) by dupling the Terms of the Ration thereof, into 2 *Hemitones*; 18 to 17, and 17 to 16. And thoſe again, by the ſame way, each into two *Dieſes*; 36 to 35, 35 to 34; the Diviſion of 18 to 17, the leſs *Hemitone*: And 34 to 33, and 33 to 32; the parts of 17 to 16, the greater *Hemitone*. But yet, none of theſe were the Complement of two *Seſquioctave Tones* to *Diateſſaron*: but another *Hemitone*, whoſe Ratio is about 20 to 19; not exactly, but ſo near it, that the Difference is only 1216 to 1215; both which together make the *Limma Pythagoricum*.

But I no where find, that they thus divided the 5th, and 3d. *Major*, but rather ſeemed to diſlike this way, becauſe of the Inequality of the *Hemitones* and *Dieſes* thus found out; and choſe rather

to

to Conſtitute their Degrees by the *Seſquioctave Tone*, and thoſe Duodecimal ſuppoſed Equal Diviſions of it. But to return.

There are, you ſee, 3 Degrees *Diatonic*; viz. *Hemitone Major*, *Tone Minor*, and *Tone Major*. The Firſt of theſe, ſome call *Degree Minor*; the Second, *Degree Major*; and the Third, *Degree Maxim*. Now theſe three Sorts of Degrees are properly to be intermixed, and ordered, in every Aſcent to an Eighth, in relation to the Key, or Uniſon given, and to the Affections of that Key, as to Flat, and Sharp, in our Scale of Muſic; ſo, that the Concords may be all true, and ſtand in their own ſettled Ration. Wherefore if you change the Key, they muſt be changed too; which is the Reaſon why a Harpſichord, whoſe Degrees are fixed; or a fretted Inſtrument, the fretts remaining fixt, cannot at once be ſet in Tune for all Keys: For, if you change the Key, you withall



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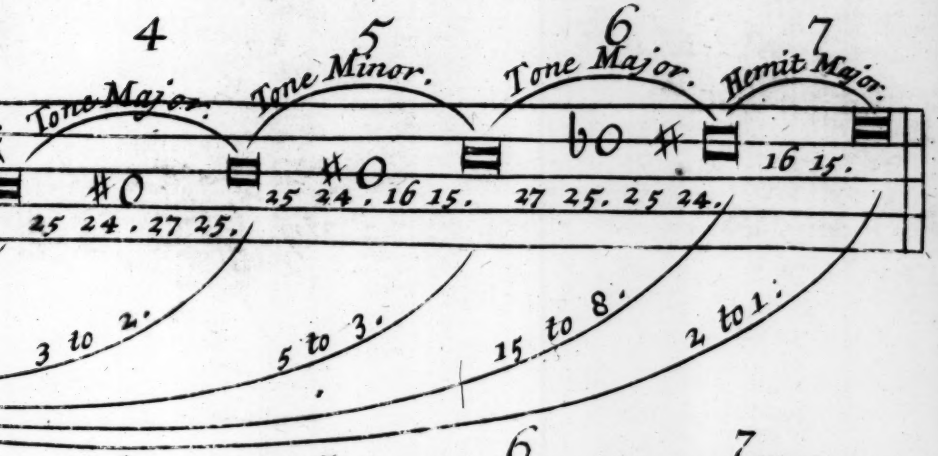
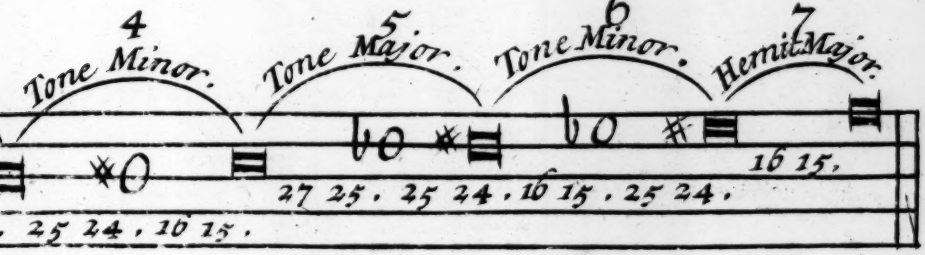
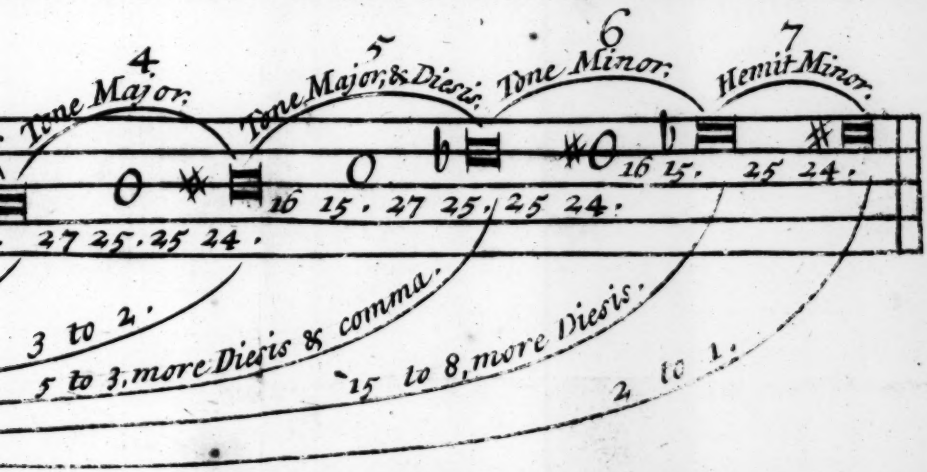

4
5
6
7
Tone Major.
Tone Minor.
Tone Major.
Hemit Major.
25 24. 27 25.
25 24. 16 15.
27 25. 25 24.
16 15.
3 to 2.
5 to 3.
15 to 8.
2 to 1.

4
5
6
7
Tone Minor.
Hemit Max.
Tone Minor.
Tone Minor.
16 15.
27 25. 25 24.
25 24.
16 15.
25 24. 16 15.


4
5
6
7
Tone Minor.
Tone Major.
Tone Minor.
Hemit Major.
25 24. 16 15.
27 25. 25 24. 16 15. 25 24.
16 15.


4
5
6
7
Tone Major.
Tone Major, & Diesis.
Tone Minor.
Hemit Minor.
27 25. 25 24.
16 15. 27 25. 25 24.
16 15.
25 24.
3 to 2.
5 to 3, more Diesis & comma.
15 to 8, more Diesis.
2 to 1.

withall change the place of *Tone Minor*, and *Tone Major*, and fall into other *Hemitones* that are not proper *Diatonic* Degrees, and consequently into false Intervals.

You may fully see this, if you draw Scales of Ascent fitted to several Keys (as are here inserted) and compare them. For an Example of this, Take the First Scale of Ascent to *Diapason* (1) *viz.* upon C Key Proper, by *Diatonic* Degrees; (making the first to be *Tone Minor*, as convenient for this Instance) intermixing the *Chromatic* and other *Hemitones*, as they are usually placed in the Keys of an Organ; *i. e.* run up an Eighth upon an Organ (tuned as well as you can) by half Notes, beginning at C *Sol fa ut*, and you will find these Measures. The Proper Degrees standing right, as they ought to be, being described by *Breves*; the other by *Semibreves*: The *Breves* representing the *Tones* of the broad Gradual Keys of an Organ; the *Semibreves* representing

preſenting the Narrow Upper Keys, which are uſually called *Muſics*. And let this be the firſt Scale, and a Standard to the reſt.

Then draw a Second Scale (II) running up an Eighth in like manner; but let the Key, or Firſt Note, be D. *Sol re* Proper, on the ſame Organ ſtanding tuned as before; which Key is ſet a Note (or *Tone Minor*) higher than the former.

Draw alſo a Third Scale (III) for *D Sol re* Key with Sharps, *viz.* Third, and Seventh, *Major*; *i. e.* F, and C, Sharp.

In the Firſt of theſe Scales, the Degrees (expreſſed by *Breves*) are ſet in good and natural Order.

In the Second Scale (changing the Key from C to D), you will find the Fourth, and Sixth, a *Comma* (81 to 80) too much; but between the Fourth, and Fifth, a *Tone Minor*, which ſhould be always a *Tone Major*. So from the Fourth

to

to the Eighth, is a *Comma* ſhort of *Diapente*; and from the Sixth, a *Comma* ſhort of 3d. *Minor*. And this, becauſe in this Scale, the Degrees are miſplaced.

The Third Scale makes the Second, Third, Fourth, and Sixth, from the *Uniſon*, each a *Comma* too much; and from the *Octave*, as much too little. In it, the 3d. Degree, between ♯F and G, is not the Proper *Hemitone*, but the Greateſt *Hemitone*, 27 to 25. And all this, becauſe in this Scale alſo, the Degrees are miſplaced; and there happen (as you may ſee) three *Tones Minor*, and but two *Major*: the Deficient *Comma* being added to the *Hemitone*.

I have added one Example more, of a Fourth Scale; (IV) *viz.* beginning at the Key ♯C; with the like Order of Degrees, as in the Firſt Scale (from the Note C♯) upon the ſame Inſtrument, as it ſtands tuned after the Firſt Scale. And this will raiſe the Firſt Scale half a Note higher.

In this Scale, all the *Hemitones* are of the ſame Meaſure with thoſe of the Firſt Scale reſpectively.

And the Intervals ſhould be the ſame with thoſe of the Firſt Scale; which has Third, Sixth, Seventh, *Major*.

But in this Fourth Scale, the 1ſt. Degree, from 𝄪C to ♭E, is *Tone Major*, and *Dieſis*; as being compounded of 16 to 15, and 27 to 25.

The 2d. Degree from ♭E to F, is *Tone Minor*; therefore the *Ditone*, made by theſe two Degrees, is too much by a *Dieſis*, (128 to 125) and as much too little the *Trihemitone*, from the *Ditone* to the Fifth.

The 3d. Degree, from F to 𝄪F, is a *Minor Hemitone*, 25 to 24; which, (though a wrong Degree) ſets the *Diateſſaron* right.

The 4th. Degree, from 𝄪F to 𝄪G, is *Tone Major*, and makes a true Fifth.

The 5th. Degree, from 𝄪G to ♭B, is *Tone Major*, and *Dieſis*; ſetting the *Hexachord*

chord (or Sixth) a *Dieſis* and *Comma* too much, or too High. It ought to have been *Tone Minor*.

The 6th. from ♭B to C, is *Tone Minor*; too little in that place by a *Comma*.

The 7th. from C to ♯C, is *Hemitone Minor*; too little by a *Dieſis*. And ſo, theſe two laſt Degrees are deficient by a *Dieſis* and *Comma*; which *Dieſis* and *Comma*, being Redundant (as before) in the 5th. Degree, are balanced by the Deficiency of a *Comma* in the 6th. Degree; and of a *Dieſis* in the 7th: And ſo the *Octave* is ſet right.

Theſe Diſagreements may be better viewed, if we ſet together, and compare the Degrees of this IV Scale, and thoſe of the I: where we ſhall find, no one of all the 7 Degrees, to be the ſame in both Scales.

Degrees.	Scale I.	Scale IV.
1ſt.	*Tone Minor.*	*Tone Maj.& Dieſis.*
2d.	*Tone Major.*	*Tone Minor.*
3d.	*Hemit. Major.*	*Hemitone Minor.*
4th.	*Tone Major.*	*Tone Minor.*
5th.	*Tone Major.*	*Tone Maj.& Dieſis.*
6th.	*Tone Major.*	*Tone Minor.*
7th.	*Hemit. Major.*	*Hemitone Minor.*

And thus it will ſucceed in all Inſtruments, Tuned in order by *Hemitones*, which are fixed upon Strings; as Harp, *&c.* Or Strings with Keys; as Organ, Harpſichord, *&c.* Or diſtinguiſhed by Fretts; as Lute, Viol, *&c.* For which there is no Remedy, but by ſome alterations of the Tune of the Strings, in the Two former; and of the Space of the Fretts in the latter; as your preſent Key will require, when you change from one Key to another, in performing Muſical Compoſitions.

Though

Though the Voice, in Singing, being Free, is naturally Guided to avoid and correct those before described *Anomalies*, and to move in the true and proper Intervals: It being much easier with the Voice to hit upon the Right, than upon the *Anomalous* or Wrong Spaces.

Much more of this Nature may be found, if you make and compare more Scales from other Keys. You will still find, that, by changing the Key, you do withall change and displace the Degrees, and make use of Improper Degrees, and produce Incongruous Intervals.

For instead of the Proper *Hemitone*, some of the Degrees will be made of other sort of *Hemitones*; amongst which chiefly are these two: *viz.* *Hemitone Maxim.* 27 to 25; and *Hemitone Minor*, or *Chromatic*, 25 to 24. Which *Hemitones* constitute and divide the two *Tones*; viz. *Tone Major*, 9 to 8: the

Terms whereof Tripled, are 27 to 24; and give 27 to 25, and 25 to 24. The *Tone Minor* likewise is divided into two *Hemitones*: viz. *Major*, 16 to 15; and *Minor*, 25 to 24.

These two serve to measure the *Tones*, and are used also, when you Divert into the *Chromatic* kind. But the *Hemitone*-Degree in the *Diatonic Genus*, ought always to be *Hemitone Major*, 16 to 15; as being the Proper Degree and Difference between *Tone Major* and *Trihemitone*, between *Ditone* and a Fourth, between Fifth and Sixth *Minor*, between Sixth *Major* and Seventh *Minor*, and also between Seventh *Major* and *Octave*.

Musick would have seem'd much Easier, if the Progression of Dividing had reached the *Hemitones*: I mean, If, as by Dupling the Terms of *Diapason*, 4 to 2; it Divides in 4 to 3, and 3 to 2; *Diatessaron*, and *Diapente*: And the Terms of *Diapente* dupled, 6 to 4; fall into 6 to 5,

and

and 5 to 4, Third *Minor*, and Third *Major*; And *Ditone*, or Third *Major*, ſo Dupled, 10 to 8, falls into 10 to 9, and 9 to 8, *Tone Minor*, and *Tone Major*: If, I ſay, in like manner, the dupled Terms of *Tone Major* 18 to 16, thus divided, had given Uſefull and Proper *Hemitones* 18 to 17, and 17 to 16. But there are no ſuch *Hemitones* found in Harmony, and we are put to ſeek the *Hemitones* out of the Differences of Other Intervals; as we ſhall have more Occaſion to ſee, when I come to treat of Differences, in *Chap.* 8.

I may conclude this Chapter, by ſhewing, how All Conſonants, and other Concinnous Intervals, are Compounded of theſe three Degrees: *Tone Major*, *Tone Minor*, and *Hemitone Major*; being ſeverally placed, as the Key ſhall require.

Tone Major, and *Hemitone Major*, joyn'd, make 3d. *Minor*.

Tone Major, and *Tone Minor*, joyn'd, make 3d. *Major*.

Tone Major, and *Tone Minor*, and *Hemitone Major*, joyn'd, make 4th.

2 *Tones Major*, 1 *Tone Minor*, 1 *Hemitone Maj.* joyn'd, make 5th.

2 *Tones Major*, 1 *Tone Minor*, 2 *Hemitones Maj.* joyn'd, make 6th. *Minor*.

2 *Tones Major*, 2 *Tones Minor*, 1 *Hemitone Maj.* joyn'd, make 6th. *Major*.

3 *Tones Major*, 1 *Tone Minor*, & 2 *Hemitones Maj.* joyn'd, make 7th. *Minor*,

3 *Tones Major*, 2 *Tones Minor*, 1 *Hemitone Maj.* joyn'd, make 7th. *Major*.

3 *Tones*

3 *Tones Major*, 2 *Tones Minor*, 2 *Hemit. Major*,	joyn'd, make	*Diapason.*
2 *Tones Major*, 1 *Tone Minor*,	joyn'd, make	*Tritone*, or falſe 4th.
1 *Tone Major*, 1 *Tone Minor*, 2 *Hemit. Major*,	joyn'd, make	*Semidiapente*, or falſe 5th.

CHAP.

CHAP. VII.

Of Diſcords.

BEſides the Degrees, which, though they conſtitute and compound all Concords, yet are reckoned amongſt Diſcords; becauſe every Degree is Diſcord to each Chord, to, or from which it is a Degree, either Aſcending, or Deſcending, as being a Second to it. Beſides theſe I ſay, there are other Diſcords, ſome greater and ſome leſs. The leſs will be found amongſt the Differences in the next Chapter; and are fit, rather to be known as Differences, than to be uſed as Intervals.

The greater Diſcords are generally made of ſuch Concords, as, by reaſon of miſplaced Degrees, happen to have a *Comma*, or *Dieſis*, or ſometimes a *Hemitone*

mitone too much, or too little: And ſo become Diſcords, moſt of them being of little uſe; only to know them, for the better Meaſuring, and Rectifying the *Syſtems*. Yet they are found amongſt the Scales of our Muſic.

Sometimes a *Tone Major* being where a *Tone Minor* ſhould have been placed, or a *Tone Minor* inſtead of a *Tone Major*; ſometime other *Hemitones*, getting the place of the *Diatonic Hemitone Major*, and ſerving for a Degree, create unapt Diſcording Intervals: amongſt which may be found at leaſt two more Seconds, two more Thirds, two more Sixths, and two more Sevenths. In each of which, one is leſs, and the other greater, than the true Legitimate Intervals, or Spaces of thoſe Denominations; as will be more explained in the enſuing Diſcourſe.

But beſides theſe (or rather amongſt them, for I here treat of Degrees as Diſcords) there are two Diſcords eminently

nently considerable, *viz. Tritone*, and *Semidiapente*. The *Tritone*, (or False Fourth) whose Ration is 45 to 32, consists of 3 whole Notes; *viz.* 2 *Tones Major*, and 1 *Minor*. The *Semidiapente*, (or False Fifth) 64 to 45; is compounded of a Fourth, and *Hemitone Major*.

And these two divide *Diapason*, 64 to 32, by the Mediety of 45; And they divide it so near to Equality, that in Practice, they are hardly to be distinguished, and may almost pass for one and the same: but in Nature, they are sufficiently distinguished, as may be seen, both by their several Rations, and several Compounding parts.

I think we may reckon Sevenths for Degrees, as well as among the greater Discording Intervals; because they are but *Seconds* from the *Octave*, and are as truly Degrees Descending, as the Seconds are in Ascent: though they be great Intervals in respect of the *Unison*, and as such may be here regarded. These

These Discords, the *Tritone*, and *Semidiapente*; as also, the Seconds, and Sevenths, are of very great use in Music, and add a wonderfull Ornament and Pleasure to it, if they be judiciously managed. Without them, Music would be much less gratefull; like as Meat would be to the Palate without Salt or Sawce. But, the further Consideration of this, and to give Directions when, and how to use them; is not my Task, but must be left to the Masters of Composition.

Discords then, such as are more apt and usefull, (*Intervalla Concinna*) are these which follow.

2d. *Minor*;	or, *Hemitone Major*,	16 to 15.
2d. *Major*;	*Tone Minor*,	10 to 9.
2d. *Greatest*;	*Tone Major*,	9 to 8.
7th. *Minor*;	5th. & 3d. *Minor*,	9 to 5.
7th. *Major*;	5th. & 3d. *Major*,	15 to 8.
Tritone;	3d. *Maj.* & *Tone Maj.*	45 to 32.
Semidiapente;	4th. & *Hemit. Maj.*	64 to 45.

These

These are the Simple Dissonant apt Intervals within *Diapason*; if you go a further compass, you do but repeat the same Intervals added to *Diapason*, or *Dis-diapason*, or *Tris-diapason*, &c. as, *Ex. gra.* a

9th. is *Diapason* with a 2d.
10th. *Diapason* with a 3d.
11th. { *Diapason* with a 4th. or, *Diapason cum Diatessaron.*
12th. { *Diapason* with a 5th. or, *Diapason cum Diapente.*
15th. *Dis-diapason.*
19th. *Dis-diapason cum Diapente.*
22th. *Tris-diapason.* &c.

Here, by the way, the Reader may take a little Diversion, in practising to measure the Rations of some of those Intervals, in the foregoing Catalogue of Discords; by comparing them with *Diapason*;

Diapaſon; as thoſe of the Sevenths, which I ſelect, becauſe they are the moſt diſtant Rations under *Diapaſon*: *Viz.* Seventh *Minor*, 9 to 5; and Seventh *Major*, 15 to 8. Now to find what Degree or Interval lies between theſe and *Diapaſon*.

Firſt, 9 to 5 is 10 to 5, wanting 10 to 9 (*Tone Minor.*) Next 15 to 8 is 16 to 8, wanting 16 to 15 (*Hemitone Major.*) So the Degree between Seventh *Minor* and *Diapaſon*, is *Tone Minor*; and between Seventh *Major* and *Diapaſon*, is *Hemitone Major*.

Then he may exerciſe himſelf in a Survey of what Intervals are comprized in thoſe ſeveral Sevenths, and of which they are compounded.

Firſt, 9 to 5 comprizeth 9 to 8, and 8 to 5: Or 9 to 8, 8 to 6, and 6 to 5. Next, 15 to 8 contain 15 to 12, 12 to 10, 10 to 9, and 9 to 8: Or 15 to 12, and 12 to 8; Or 15 to 10, and 10 to 8, *&c.* I ſuppoſe, that the

Reader

Reader, before this, is ſo perfect in theſe Rations, that I need not loſe time to name the Intervals expreſſed by the Mean Rations, contained in the foregoing Rations of the Sevenths, which ſhew of what Intervals the ſeveral Sevenths are compounded.

Beſides theſe, (by reaſon of Degrees wrong placed) there are two more 7ths. [falſe 7ths] one, leſs than the true ones; and another greater. The leaſt compounded of two Fourths; whoſe Ration is 16 to 9, and wants a *Comma* of 7th. *Minor*, and a *Tone Major* of *Diapaſon*: The other is the greateſt, called *Semidiapaſon*, whoſe Ration is 48 to 25; being a *Dieſis* more than 7th. *Major*; and wanting *Hemitone Minor* of *Diapaſon*.

Now firſt, 16 to 9, is 16 to 8 (2 to 1) wanting 9 to 8; *i. e.* wanting *Tone Major* of *Diapaſon*; and contains 16 to 10 (8 to 5) and 10 to 9. Or, 16 to 15, 15 to 12 (5 to 4) 12 to 10 (6 to 5) and

and 10 to 9. Next, *Semi-diapaſon* 48 to 25, is 50 to 25, wanting 50 to 48; *i.e.* 25 to 24 (viz. *Hemitone Minor*) of *Diapaſon.*

And the like happens, as hath been ſaid, to the other Intervals, which admit of *Major* and *Minor*; viz. 2ds, 3ds, and 6ths. The 4th, and 5th, and 8th, ought always to remain immutable; though they may ſuffer too ſometimes, and incline to Diſcord, if we aſcend to them by very wrong Degrees; as you may ſee in the II Scale in the foregoing Chapter: where the Fourth having 2 *Tones Major*, is a *Comma* too much.

All theſe Intervals may be ſubject to more Mutations, by more abſurd placing of Degrees, or of Differences of Degrees; but it is not worth the Curioſity to ſearch farther into them. The Reader may take pleaſure, and ſufficiently Exerciſe himſelf in comparing and meaſuring theſe which are already laid before him.

But to return from this Digreſſion. There are many unapt Diſcords, which may ariſe by continual Progreſſion of the ſame Concords; *i. e.* by Adding (for Example) a 4th. to a 4th, a 5th. to a 5th, *&c.* For it is obſervable, That, onely *Diapaſon* added (as oft as you pleaſe) to *Diapaſon*, ſtill makes Concord: But any other Concord, added to it ſelf, makes Diſcord.

You will ſee the Reaſon of it, when you have conſidered well the Anatomy (as I may call it) of the Conſtitutive parts of *Diapaſon*; which contains, and is compoſed of 7 Spaces of Degrees, or of 4th. and 5th, or of 3ds. and 6ths, or of 2ds. and 7ths; which muſt all keep their true Meaſures and Rations belonging to them, and otherwiſe are eaſily and often diſordered.

Then, conſider *Diapaſon* as conſtituted of two Fourths Disjunct, and a *Tone Major* between them. And this laſt is moſt needfull to be very well conſidered;

dered; as moſt plainly ſhewing the Reaſons of thoſe Anomalies, or Irregular Intervals, which are produced by Changing the Key; and conſequently giving a new and wrong place to this Odd *Tone Major*, which ſtands in the midſt of *Diapaſon*, between the two Fourths Disjunct.

Every 4th. muſt conſiſt of one *Tone Major*, one *Tone Minor*, and one *Hemitone Major*, as its Degrees, placing them in what Order you pleaſe; whoſe Rations, added together, make the Ration of *Diateſſaron*. And of theſe ſame Degrees contained in the 4th, are made the two 3ds, which conſtitute the 5th. *Tone Major* and *Hemitone Major* make the leſs 3d, or *Trihemitone*; *Tone Major* and *Tone Minor* make the Greater 3d, or *Ditone*; *Trihemitone* and *Ditone* make *Diapente*; *Trihemitone* and *Tone Minor* (as likewiſe *Ditone* and *Hemitone Major*) make *Diateſſaron*.

Now this *Tone Major*, that ſtands in the middle of *Diapaſon*, between the two 4ths, which it disjoins; and the Degrees required to the 4ths, will not in a fixed Scale ſtand right, when you alter your Key, and begin your Scale of *Diapaſon* from another Note. For that which was the 5th, will now be the 4th, or 6th, *&c.* And then the Degrees will be diſordered, and create ſome diſcording Intervals. If you continue conjunct Fourths, there will be a Defect of *Tones Major*; if you continue conjunct Fifths, there will be too many *Tones Major* in the Syſtems produced. And if a *Tone Major* be found, where it ought to have been a *Tone Minor*; or a *Minor* inſtead of a *Major*; that Interval will have a *Comma* too much, or too little. And ſo likewiſe will from a wrong *Hemitone* be found the Difference of a *Dieſis*. And theſe two, *Comma* and *Dieſis*, are ſo often Redundant, or Deficient, according as the Degrees happen to be diſordered or miſ-placed; that

that thereby, the Difficulties of fixing half Notes of an Organ in tune for all Keys, or giving the true Tune by Fretts, become ſo Inſuperable.

You ſee, that in every Space of an Eighth, there are to be 3 *Tones Major*, and 2 *Tones Minor*, and 2 *Hemitones Major*: One *Tone Major* between the *Diateſſaron* and *Diapente*, and a *Tone Major*, a *Tone Minor*, and *Hemitone Major* in each of the Disjunct Fourths.

Theſe are the Proper Degrees by which you ſhould always Aſcend or Deſcend through *Diapaſon*, in the *Diatonic* Kind; which *Diapaſon*, being the compleat Syſtem, containing all primary Simple *Harmonic* Intervals that are; (and for that reaſon called *Diapaſon*;) You may multiply it, or add it to its ſelf as oft as you pleaſe, as far as Voice or Inſtrument can reach; and it will ſtill be Concord, and cannot be diſordered by ſuch addition: becauſe every of them will contain (however placed) juſt

3 *Tones*

3 *Tones Major*, 2 *Tones Minor*, and 2 *Hemitones Major*.

Whereas, if you add any other Interval to it ſelf, the Degrees will not fall right, and it will be Diſcord, becauſe all Concords are compounded of unequal Parts, as hath been ſhew'n before; and if you carry them in Equal Progreſſion, they will mix with other Intervals by incongruous Degrees, and thoſe Diſordered Degrees will create a Diſſonant Interval. See the following Scheme of it.

2 3ds. *Minor*	make	5th, wanting *Hem. Min.*
2 3ds. *Major*		5th, and *Hemit. Minor.*
2 4ths.		8th, wanting *Tone Maj.*
2 5ths.		8th, and *Tone Major.*
2 6ths. *Minor*		8th, and *Ditone*, & *Dieſis.*
2 6ths. *Major*		8th, & 4th, & *Hem. Min.*

To which may be added, That

2 *Tones Min.*	make	*Ditone*, wanting a *Comma.*
2 *Tones Maj.*		*Ditone*, and a *Comma.*

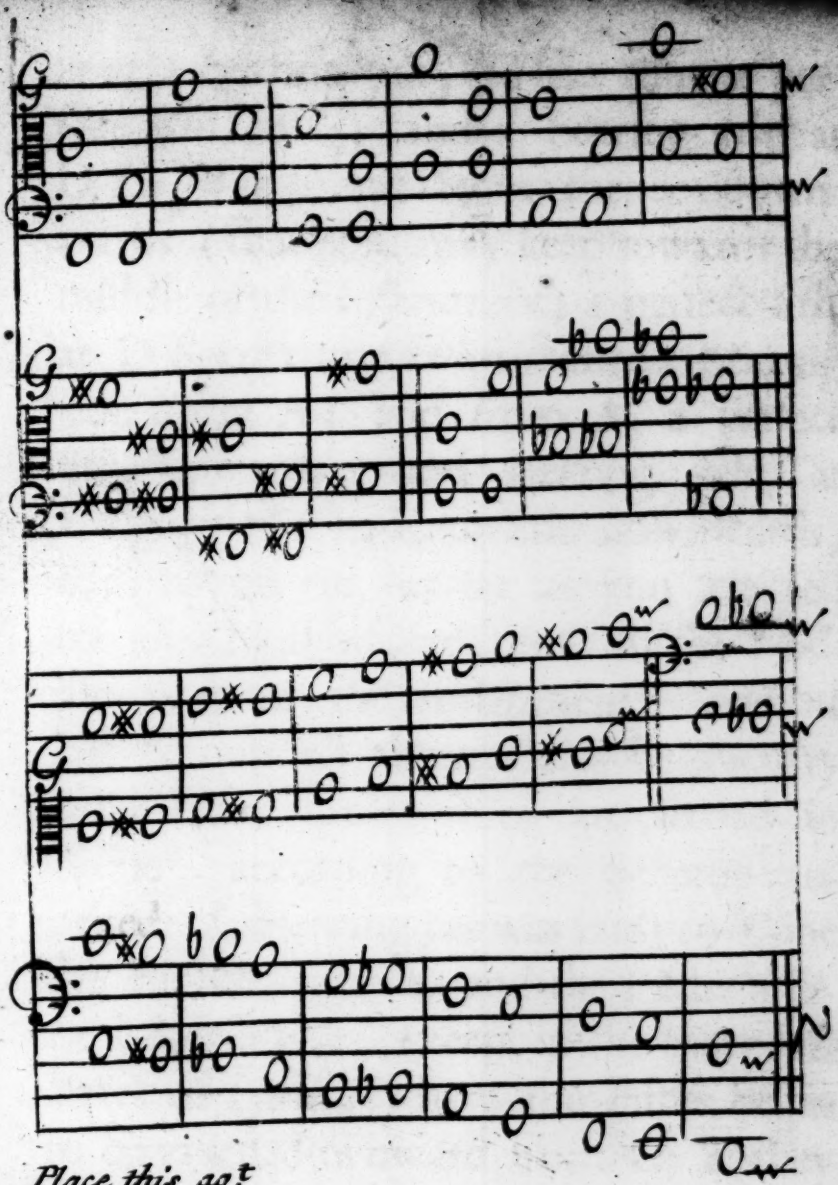
It

It was ſaid above, that *Diapaſon* may be added to it ſelf as oft as you pleaſe, and there will be no diſorder, becauſe every one of them will ſtill retain the ſame Degrees of which the firſt was Compoſed: But it is not ſo in other Concords; of which I will add one more Example, becauſe of the uſe which may be made of it.

Make a Progreſſion of 4 *Diapente's*, and, as was ſhewed in the 5th. Chapter, (*Pag.* 102.) it will produce *Diſ-diapaſon*, and 2 *Tones Major*, which is a 17th, with a *Comma* too much. Becauſe in that Space there ought to be juſt 7 *Tones Major*, and 5 *Tones Minor*; Whereas in 4 Fifths continued, there will be found 8 *Tones Major*, and but 4 *Tones Minor*: So that a *Tone Major*, getting the place of a *Tone Minor*, there will be in the whole Syſtem a *Comma* too much. One of theſe *Major Tones* ſhould have been a *Tone Minor*, to make the Exceſs above *Diſ-diapaſon* a juſt *Ditone*.

On the other ſide, if you continue the Ration of 4 *Diateſſarons*, there will be a *Tone Minor*, inſtead of a *Tone Major*; and conſequently a *Comma* deficient in conſtituting *Diapaſon* and 6th. *Minor*. For ſince every Fourth muſt conſiſt of the Degrees of *Tone Minor*, one *Tone Major*, one *Hemitone Major*; it follows, that if you continue 4 Fourths, there will be 4 *Tones Minor*, 4 *Tones Major*, and 4 *Hemitones Major*. Whereas in the Interval of *Diapaſon* with 6th. *Minor*, there ought to be 5 *Tones Major*, and but 3 *Minor*.

By this you may ſee the Reaſon, why, to put an Organ or Harpſichord into more general uſefull Tune, you muſt tune by 8ths, and 5ths; making the 8ths. perfect, and the 5ths. a little bearing downward; *i. e.* as much as a quarter of a *Comma*, which the Ear will bear with in a 5th, though not in an 8th. For Example, begin at C *Fa ut*; make C *Sol fa ut* a perfect 8th. to it, and G *Sol re ut*



Place this ag.t
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re ut a bearing 5th; Then tune a perfect 8th. to G, and a bearing 5th. at D *La ſol re*; and from thence downwards (that you may keep towards the middle of the Inſtrument) a perfect 8th. at D *Sol re*: and from thence a bearing 5th. up at A; and from A, a perfect 8th. upwards, and bearing 5th. at E *La mi*. From E an 8th. downwards; and ſo go on, as far as you are led by this Method, to tune all the Middle part of the Inſtrument: and at laſt fill up all above, and below, by 8ths. from thoſe which are ſettled in Tune; according to the Scheme annexed. Obſerving (as was ſaid) to Tune the Eighths perfect, and the Fifths a little bearing Flat; except in the three laſt Barrs of Fifths, where the Fifths begin to be taken downward from C, as they were upwards in all before: Therefore, as before, the Fifth above bore downward; ſo here, the Fifth below muſt bear upward, to make a Bearing Fifth; but

That

That being not ſo eaſie to be judged, Alter the Note below, till you judge the Note above to be a Bearing Fifth to it. This will rectifie both thoſe Anomalies of 5ths and 4ths. For the 5th. to the *Uniſon*, is a 4th. to the *Octave*; and what the 5th. looſeth by Abatement, the 4th. will gain: which doth in a good Degree rectifie the Scale of the Inſtrument. Taking care withall, that what Anomalies will ſtill be found in this *Hemitonic* Scale, may, by the Judgment of your Ear, in Tuning, be thrown upon ſuch Chords as are leaſt uſed for the Key: as ♯G, ♭E, *&c.* Even which the Ear will bear with, as it doth with other Diſcords in binding paſſages; if ſo, you cloſe not upon them. But the other Diſcords, ſo uſed, are moſt Elegant; theſe only more Tolerable.

CHAP.

CHAP. VIII.

Of Differences.

ALL Rations and Proportions of inequality, have a Difference between them, when compared to one another; and conſequently the Intervals, expreſſed by thoſe Rations, differ likewiſe. A Fifth is Different from a Fourth, by a *Tone Major*; from a 3d. *Minor*, by a 3d. *Major*; ſo an 8th. from a 5th, by a 4th. Of the Compounding parts of any Interval, one of them is the Difference between the other part and the whole Interval.

But I treat now of ſuch Differences as are generally leſs than a *Tone*, and create the Difficulties, and Anomalies occurring in the two foregoing Chapters. I have the leſs to ſay of them a-part,

a-part, becauſe I could not avoid touching upon them all along. It will onely therefore be needfull, to ſet before you an orderly view of them. And firſt, taking an account of the true *Harmonic* Intervals with their Differences, and the Degrees by which they ariſe; it will be eaſier to judge of the falſe Intervals, and of what concern they are to Harmony.

Table

Table of true *Diatonic* Intervals within *Diapaſon*, with the Differences between them.

			Their Rations.	Their Differences.
Hemitone Major.			16 to 15	25 to 24
Tone Minor.			10 to 9	81 to 80
Tone Major.			9 to 8	16 to 15
3d. *Minor*;	Compounded of	*Tone Maj.* & *Hemit. Maj.*	6 to 5	25 to 24
3d. *Major*;		*Tone Major* & *Tone Minor.*	5 to 4	
4th.		3d. *Minor* & *Tone Min.* or 3d. *Major* & *Hemitone Major.*	4 to 3	10 to 9 9 to 8
5th.		4th, and *Tone Major*; or of the two 3ds.	3 to 2	
6th. *Minor*;		5th, and *Hemit. Major*; or 4th, and 3d. *Min.*	8 to 5	16 to 15
6th. *Major*;		5th, and *Tone Minor*; or 4th, and 3d. *Major.*	5 to 3	25 to 24
7th. *Minor*;		6th. *Maj.* & *Hem. Max.* or 6th. *Minor* & *Tone Major*; or 5th, and 3d. *Minor.*	9 to 5	27 to 25 25 to 24
7th. *Major*;		6th. *Major*, & *Tone Maj.* or 5th, & 3d. *Major.*	15 to 8	16 to 15
Diapaſon;		7th, and 2d. or 6th, & 3d. or 5th, & 4th.	2 to 1	
Tritone;		3d. *Major*, and *Tone Maj.*	45 to 32	2048 to 2025
Semidiapente;		4th, and *Hemit. Major.*	64 to 45	

Thoſe which ariſe from the Differences of Conſonant Intervals, are called *Intervalla Concinna*, and properly appertain to Harmony: The reſt are neceſſary to be known, for making and underſtanding the Scales of Muſick.

Table

Table of falſe *Diatonic* Intervals, cauſed by Improper Degrees; with their Rations and Differences from the true Intervals.

This Mark + ſtands for *More*; — for *Leſs*.

			Rations.	Differences from the true.
Tribemitone	Leaſt;	*Tone Minor*, and *Hemit. Major.*	32 to 27	81 to 80 —
	Greateſt;	*Tone Major*, and *Hemit. Max.*	243 to 200	81 to 80 +
Ditone	Leaſt;	2 *Tones Minor.*	100 to 81	81 to 80 —
	Greateſt;	2 *Tones Major.*	81 to 64	81 to 80 +
Fourth	Leſs;	2 *Tones Minor*, & *Hemit. Major.*	320 to 243	81 to 80 —
	Greater;	2 *Tones Major*, & *Hemit. Major.*	27 to 20	81 to 80 +
Fifth	Leſs;	Leſs 4th, and *Tone Major.*	40 to 27	81 to 80 —
	Greater;	Greater 4th, & *Tone Major.*	243 to 160	81 to 80 +
Sixth	Leaſt;	5th, and *Hemit. Minor.*	25 to 16	81 to 80 —
	Greateſt;	5th, and *Tone Major.*	27 to 16	81 to 80 +
Seventh	Leaſt;	6th. *Major*, and *Hemit. Major.*	16 to 9	81 to 80 —
	Greateſt;	6th. *Minor*, and 3d. *Minor.*	48 to 25	128 to 125 +

Here

Here in this account may be ſeen, how frequently the *Comma*, and the *Dieſis* Abounding, or Deficient, by reaſon of Miſ-placed Degrees, occaſion Diſcord in *Harmonic* Intervals.

The *Comma*, by reaſon of a wrong *Tone*, *i.e.* too much; when a *Tone Major* happens where there ought to be a *Tone Minor*: or too little, when the *Tone Minor* is placed inſtead of the *Major*. And the *Dieſis* is Redundant, or Deficient, by reaſon of a wrong *Hemitone*; when the *Major* happens inſtead of the *Minor*, or the Contrary: the *Dieſis* being the Difference between them. And if *Hemitonium Maximum* get in the place of *Hemitonium Majus*, the Exceſs will be a *Comma*; if in the place of *Hemitone Minor*, the Exceſs will be *Comma* and *Dieſis*.

And theſe Anomalies are not Imaginary, or only Poſſible, but are Real in an Inſtrument fixed in Tune by *Hemitones*; as, *Organ*, *Harpſichord*, &c. And the Reader may find ſome of them amongſt

mongſt thoſe 4 Scales of *Diapaſon*, in the 6th. Chapter; to which alſo more may be added: Out of the Firſt of which, I have Selected ſome Examples; uſing the common Marks, as before: *viz.* +, for *More*; and — for *Leſs*, or *Wanting*.

From
- ♯C, to ♭E; { *Tone Major*, + *Dieſis*; or, 3d. *Min.* — *Hemit. Min.*
- ♯C, to F; { 3d. *Major*, + *Dieſis*; or, 4th, — *Hemit. Minor.*
- D, to G; 4th, + *Comma.*
- ♭E, to ♯F; { 3d *Min.* — *Dieſ.* & *Com.* or *Tone Min.* + *Hem. Min.*
- ♭E, to ♯G; { 4th, — *Dieſis*; or, 3d. *Maj.* + *Hem. Min.*
- ♯F, to ♭B; { 3d. *Maj.* + *Dieſ.* & *Com.* or 4th, — *Hem. Subminim.*
- ♯F, to B; 4th, + *Comma.*
- ♯G, to ♭B; { *Tone Maj.* + *Dieſis*; or, 3d. *Min.* — *Hemit. Min.*
- ♯G, to C; { 3d. *Maj.* + *Dieſis*; or, 4th, — *Hemit. Min.*
- B, to D; 3d. *Min.* — *Comma.*

Next,

Next, take account of ſome Differences which conſtitute ſeveral *Hemitones*.

Difference between

Tone Major, and *Hemit. Minor.*	*Hem. Maxim.*	27 to 25
3*d. Major*, and 4*th.*	*Hemit. Majus:*	16 to 15
Tone Major, and *Hemit. Major.*	*Hemitone Medium.*	135 to 128
3*d. Minor*, and 3*d. Maj.*	*Hem. Min. Diesis Chromat.*	25 to 24
2 *Tones Major*, and 4*th.*	*Hemiton.* (or *Limma*) *Pythagor.*	256 to 243
Tone Maj. and *Limma.*	*Apotome* or *Hemit. Med.* with *Comma.*	2187 to 2048

To which may be added, out of *Merſennus*.

Differ. between

Hemit. Maxim. and *Hemit Minor.*	*Hemitonium. Minimum.*	648 to 625
Tone Minor, and *Hemitone Maxim.* or, *Hemitone Minor*, and *Comma.*	*Hemitonium. Subminimum.*	250 to 243

Next, take a farther View of Differences, moſt of which ariſe out of the preceding Differences, by which you

will better see how all Intervals are Compounded, and Differenced; and more easily judge of their Measures.

Table of more Differences.

Difference between

Tone Maj. and *Tone Min.*	*Comma.*
Tone Maj. and *Hem. Greatest.*	*Hemitone Minor.*
Tone Maj. & *Hem. Medium.*	*Hemitone Majer.*
Tone Maj. and *Hem. Pythag.*	*Apotome.*
Hem. Greatest, and *Hem. Maj.*	*Comma.*
Hem. Greatest, & *Hem. Min.*	*Comma*, and *Diesis*; viz. *Hem. Minimum.*
Hemit. Major, and *Minor.*	*Diesis.*
Hemit. Major, and *Medium.*	2048 to 2025. viz. *Comma Minus.*
Hemit. Major, and *Pythag.*	*Comma.*
Apotome, and *Hemit. Majus.*	Diff. betw. *Comma Majus*, and *Min.*
Apotome, and *Hemit. Med.*	*Comma.*
Apotome, and *Hem. Pythag.*	*Comma*, and the aforesaid Differ.
Apotome, and *Hemit. Minus.*	2 *Comma's.*
Hemit. Medium, and *Pythag.*	Differ. of *Comma Majus*, and *Minus.*
Hemit. Medium, and *Minus.*	*Comma.*
Hemit. Pythag. and *Minus.*	*Comma Minus.*
Hemit. Minus, and *Diesis.*	Somewhat more than *Comma*, viz. 3125 to 3072.
Hemit. Minus, and *Comma.*	*Hem. Subminimum.*
Diesis, and *Comma.*	*Comma Minus*, viz. 2048 to 2025.
Com. Majus, and *Com. Minus.*	32805 to 32768.

These

These Differences (with some more) are found between several other Intervals; of which more Tables might be drawn, but I shall not trouble the Reader with them. Having here shewn what they are, he may (if he please) exercise himself to examine These by Numbers, and also find out Them; and to some it may be pleasant and delightfull; And for that reason, I have the more largely insisted on this part of my Subject, which concerns the Measures, Habitudes, and Differences of Harmonic Intervals.

I shall add one Table more; of the Parts, of which these lesser Intervals are Compounded; which will still give more Light to the former; and is, in Effect, the same.

Tone Major contains, & is compounded of { *Tone Minor*, and *Comma*. { *Hemitone Maxim.* and *Hemitone Min.* / *Hemitone Maj.* / *Hemitone Med.* { *Limma*, / *Apotome*. { 2 *Hem. Min.* / 1 *Diesis*, / 1 *Comma*.

Tone Min. { *Hem. Maxim.* / *Hem. Submin.* { *Hemit. Major*, / *Hemit. Min.* { 2 *Hemit. Min.* / 1 *Diesis*.

Hem. Max. { *Hem. Maj.* / *Comma*. { *Hem. Med.* / *Diesis*. { *Hem. Pyth.* / 2 *Comma's*. { *Hem. Min.* / *Dies. & Com.*

Hem. Maj. { *Hem. Med.* / *Com. Min.* { *Hem. Pyth.* / *Comma*. { *Hem. Min.* / *Diesis*. { *Hem. Submi.* / *Dies. & Com.*

Hem. Med. { *Hem. Min.* / *Comma*. { *Hemitone Pythagoricum.* / Difference between *Comma Majus*, and *Minus*, viz. 32805 to 32768.

Hem. Pyth. { *Hemitone Minus.* / *Comma Minus.*

Hem. Min. { *Hemit. Submin.* / *Comma*. { *Diesis*, and / 3125 to 3072.

Diesis. { *Comma*, / *Comma Minus.*

Comma. { *Comma Minus*, / 32805 to 32768.

I think there scarce needs an Apology for some of these Appellations, in respect of Grammar. That I call *Hemitonium*, and *Hexachordon*, *Majus*, and *Minus*; sometimes *Hemitone*, and *Hexachord*, *Major*, and *Minor*. These two last Words are so well adapted to our Language, that there is no *English*-man, but knows them. Therefore when I make *Hemitone* an *English* word, I take *Major*, and *Minor*, to be so too; and fittest to be joyned with it, without respect of *Gender*.

CHAP.

CHAP. IX.

Concluſion.

TO Conclude all. Bodies by Motion make Sound; Sound, of fitly Conſtituted bodies, makes Tune: Tune, by Swiftneſs of Motion is rendred more Acute; by Slowneſs more Grave: in Proportion to the Meaſure of Courſes and Recourſes, of Tremblings or Vibrations of Sonorous Bodies. Thoſe Proportions are found out by the Quantity and Affections of Sounding Bodies. *Ex. gr.* by the Length of Chords. If the Proportion of Length (*Cæteris paribus*,) and conſequently of Vibrations of ſeveral Chords, be commenſurate within the Number 6; then thoſe Intervals of Tune are Conſonant, and make Concord, the Motions mix-

ing and uniting as they paſs: If incommenſurate, they make Diſcord by the jarring and claſhing of the Motions. Concords are within a limited Number, Diſcords innumerable. But of them, thoſe only here conſidered, which are (as the *Greeks* termed them) ἐμμελῆ, *Concinnous*, apt and uſefull in Harmony: Or which, at leaſt, are neceſſary to be known, as being the Differences and Meaſures of the other; and helping to diſcover the reaſon of Anomalies, found in the Degrees of Inſtruments tuned by *Hemitones*.

All theſe I have endeavoured to explain, with the manifeſt Reaſons of Conſonancy and Diſſonancy, (the Properties of a *Pendulum* giving much light to it,) ſo as to render them eaſie to be underſtood by almoſt all ſorts of Readers; and to that end have enlarged, and repeated, where I might, to the more intelligent Reader, have compriſed it very much ſhorter. But I hope

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the Reader will pardon that, which could not well be avoided, in order to a full, and clear Explanation of that, which was my deſign, *viz.* the Philoſophy of the Natural Grounds of Harmony.

Upon the Whole; You ſee how Rationally, and Naturally, all the Simple Concords, and the Two *Tones*, are found and demonſtrated, by Sub-diviſions of *Diapaſon*.

2 to 1, *i. e.* 4 to 2; into 4 to 3, and 3 to 2.
2 to 1, *i. e.* 6 to 3; into 6 to 5, and 5 to 3.
2 to 1, *i. e.* 8 to 4; into 8 to 5, and 5 to 4.
2 to 1, *i.e.* 10 to 5; into 10 to 9, 9 to 8, and 8 to 5.

In which are the Rations (in Radical, or Leaſt Numbers) of the *Octave*, Fifth, Fourth, Third *Major*, Third *Minor*, Sixth *Major*, Sixth *Minor*, and *Tone Major*, and *Tone Minor*.

And then, All the *Hemitones*, and *Dieſis*, and *Comma*, are found by the Differences of theſe, and of one another; as hath been ſhewn at large.

Now, certainly, this is much to be preferred before any Irrational Contrivance of expreſſing the ſeveral Intervals. The *Ariſtoxenian* way of dividing a *Tone* [*Major*] into 12 Parts, of which 3 made a *Dieſis*, 6 made *Hemitone*, 30 made *Diateſſaron*; (as hath been ſaid) might be uſefull, as being eaſier for Apprehenſion of the Intervals belonging to the three Kinds of Muſick; and might ſerve for a leaſt common Meaſure of all Intervals, (like Mr. *Mercator*'s artificial *Comma*) 72 of them being contained in *Diapaſon*.

But this way, and ſome other Methods of dividing Intervals equally, by *Surd* Numbers and *Fractions*, attempted by ſome Modern Authors; could never conſtitute true Intervals upon the Strings of an Inſtrument, nor afford any Reaſon for the Cauſes of Harmony, as is done by the Rational Way, explaining Conſonancy by united Motions, and Coincidence of Vibrations. And though they ſuppoſed ſuch Diviſi-

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ons of Intervals; yet we may well believe, that they could not make them, nor apply them in tuning a Musical Instrument; and if they could, the Intervals would not be true nor exact. But yet, the Voice offering at those, might more easily fall into the true Natural Intervals. *Ex. gr.* The Voice could hardly express the Antient *Ditone* of 2 *Tones Major*; but aiming at it, would readily fall into the Rational Consonant *Ditone* of 5 to 4, consisting of *Tone Major*, and *Tone Minor*. It may well be rejected as unreasonable, to measure Intervals by Irrational Nnmbers, when we can so easily discover and assign their true Rations in Numbers, that are *Minute* enough, and easie to be understood.

I did not intend to meddle with the Artificial part of Musick: The Art of Composing, and the Metric and Rhythmical parts, which give the infinite Variety of Air and Humor, and indeed the very Life to Harmony; and which

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can make *Muſick*, without Intervals of Acuteneſs and Gravity, even upon a Drum; and by which chiefly the wonderfull Effects of Muſick are performed, and the Kinds of Air diſtinguſhed; As, *Almand*, *Corant*, *Jigg*, &c. which variouſly attack the Fancy of the Hearers; ſome with Sprightfulneſs, ſome with Sadneſs, and others a middle way. Which is alſo improved by the Differences of thoſe we call Flat, or Sharp Keys; The Sharp, which take the Greater Intervals within *Diapaſon*, as 3ds, 6ths, and 7ths. *Major*; are more Brisk and Airy; and being aſſiſted with Choice of Meaſures laſt ſpoken of, do Dilate the Spirits, and Rouze them up to Gallantry, and Magnanimity. The Flat, conſiſting of all the leſs Intervals, contract and damp the Spirits, and produce Sadneſs and Melancholy. Laſtly, A mixture of theſe, with a ſuitable *Rhythmus*, gently fix the Spirits, and compoſe them in a middle Way: Wherefore the

Firſt

Firſt of theſe is called by the Greeks *Diaſtaltic*, Dilating; the ſecond, *Syſtaltic*, Contracting; the laſt, *Heſychiaſtic*, Appeaſing.

I have done what I deſigned, ſearched into the Natural Reaſons and Grounds, the Materials of Harmony; not pretending to teach the Art and Skill of Muſick, but to diſcover to the Reader the Foundations of it, and the Reaſons of the *Anomolous Phœnomena*, which occurr in the Scales of Degrees and Intervals: Which though it be enough to my Purpoſe, yet is but a ſmall (though indeed the moſt certain, and, conſequently moſt delightfull) Part of the Philoſophy of Muſick; in which there remain Infinite Curious Diſquiſitions that may be made about it: As what it is, that makes Humane Voices, even of the ſame Pitch, ſo much to differ one from another? (For though the Differences of Humane Countenances are viſible; yet we cannot ſee the Differences of Inſtruments

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of Voice, nor conſequently of the Motions and Colliſions of Air, by which the Sound is made.) What it is that conſtitutes the different Sounds of the Sorts of Muſical Inſtruments, and even ſingle Inſtruments? How the Trumpet, only by the Impulſe of Breath, falls into ſuch variety of Notes, and in the Lower Scale makes ſuch Natural Leaps into Conſonant Intervals of 3d, 4th, 5th, and 8th. But this I find is very ingeniouſly explicated by an Honourable Member of the *R. S.* and publiſhed in the *Philoſophical Tranſactions*, N° 195. Alſo how the *Tube-Marine*, or Sea-Trumpet (a *Monochord*) ſo fully expreſſeth the Trumpet; and is alſo made to render other Varieties of Sounds; as, of a Violin, and Flageolet, whereof I have been an Ear-witneſs? How the Sounds of Harmony are received by the Ear; and why ſome perſons do not love Muſick? *&c.*

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As to this laſt; the incomparable Dr. *Willis* mentions a certain Nerve in Brain, which ſome Perſons have, and ſome have not. But further, it may be conſidered, that all Nerves are compoſed of ſmall Fibres; Of ſuch in the Gutts of of Sheep, Cats, *&c.* are made Lute-Strings: And of ſuch are all the Nerves, and amongſt them, thoſe of the Ear, compoſed. And, as ſuch, the latter are affected with the Regular Tremblings of Harmonic Sounds. If a falſe String (ſuch as I have before deſcribed) tranſmit its Sound to the beſt Ear; it diſpleaſeth. Now, if there be found falſeneſs in thoſe Fibres, of which Strings are made; Why not the like in thoſe of the Auditory Nerve in ſome Perſons? And then it is no wonder if ſuch an Ear be not pleaſed with Muſick, whoſe Nerves are not fitted to correſpond with it, in commenſurate Impreſſions and Motions. I gave an Inſtance in *Chap.* 3d. how a Bell-Glaſs will Tremble and Eccho to its own Tune,

Tune, if you hit upon it: And I may add, That if the Glaſs ſhould be irregularly framed, and give an uncertain Tune, it would not anſwer your Trial. In fine, Bodies muſt be Regularly framed to make Harmonic Sounds, and the Ear Regularly conſtituted to receive them. But, this by the by; and only for a Hint of Inquiry.

I was ſaying, that there remain Infinite Curioſities relating to the Nature of Harmony, which may give the moſt Acute Philoſopher buſineſs, more than enough, to find out; and which perhaps will not appear ſo eaſie to demonſtrate and explain, as are the Natural Grounds of Conſonancy and Diſſonancy.

After all therefore, and above all, by what is alraedy diſcovered, and by what yet remains to be found out; we cannot but ſee ſufficient cauſe to Rouze up our beſt Thoughts, to Admire and Adore the Infinite Wiſdom and Goodneſs of Almighty God. His Wiſdom,

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in ordering the Nature of Harmony in ſo wonderfull a manner, that it ſurpaſſeth our Underſtanding to make a through Search into it, though, (as I ſaid) we find ſo much by Searching, as does recompence our Pains with Pleaſure, and Admiration.

And his Goodneſs, in giving Muſick for the Refreſhings and Rejoycings of Mankind; ſo that it ought, even as it relates to common Uſe, to be an Inſtrument of our great Creator's Praiſe, as he is the Founder and Donor of it.

But much more, as it is advanced and ordained to relate immediately to his Holy Worſhip, when we Sing to the Honour and Praiſe of God. It is ſo Eſſential a part of our Homage to the Divine Majeſty, that there was never any Religion in the World, *Pagan*, *Jewiſh*, *Chriſtian*, or *Mehumetan*, that did not mix ſome kind of Muſick with their Devotions; and with Divine Hymns, and Inſtruments of Muſick, ſet forth the

Honour

Honour of God, and celebrate his Praiſe. Not only, *Te decet Hymnus Deus in Sion.* (Pſal. 65.) but alſo ..... *Sing unto the Lord all the whole Earth.* (Pſal. 96.)

And it is that, which is inceſſantly performed in Heaven before the Throne of God, by a General Conſort of all the Holy Angels and the Bleſſed.

In ſhort, we are in Duty and Gratitude bound to bleſs God, for our Delightfull Refreſhments by the uſe of Muſick; But eſpecially in our publick Devotions, we are obliged by our Religion, with Sacred Hymns and Anthems, to magnifie his Holy Name; that we may at laſt find Admittance above, to bear a Part in that Bleſſed Conſort, and Eternally Sing *Allelujahs*, and *Triſagions* in Heaven.

Δόξα τῷ Θεῷ.

FINIS.